The University of Jordan

Authorization Form

I, Abdulmajeed Hamid authorize the University of Jordan to supply copies of my Thesis/ Dissertation to libraries or establishments or individuals on request, according to the University of Jordan regulations.

Signature:

Date: 2/6/2011

الجامعة الأردنيـــة
كلية الدراسات العليا

التاريـــخ: / /

نموذج رقم (١٨)
إقرار والتزام بالمعايير الأخلاقية والأمانة العلمية
وقوانين الجامعة الأردنية وأنظمتها وتعليماتها
لطلبة الماجستير

أنا الطالب: عبد الحميد حامد القليطي الرقم الجامعي: (٨٠٦١٩٤٤)
تخصص: رياضيات الكليـــة: العلوم

عنوان الرسالة:
Goodness of fit Test for The weibull distribution

اعلن بأنني قد التزمت بقوانين الجامعة الأردنية وأنظمتها وتعليماتها وقراراتها السارية المفعول المتعلقة باعداد رسائل الماجستير عندما قمت شخصيا" باعداد رسالتي وذلك بما ينسجم مع الأمانة العلمية وكافة المعايير الأخلاقية المتعارف عليها في كتابة الرسائل العلمية. كما أنني أعلن بأن رسالتي هذه غير منقولة أو مستلة من رسائل أو كتب أو أبحاث أو أي منشورات علمية تم نشرها أو تخزينها في أي وسيلة اعلامية، وتأسيسا" على ما تقدم فانني أتحمل المسؤولية بأنواعها كافة فيما لو تبين غير ذلك بما فيه حق مجلس العمداء في الجامعة الأردنية بالغاء قرار منحي الدرجة العلمية التي حصلت عليها وسحب شهادة التخرج مني بعد صدورها دون أن يكون لي أي حق في التظلم أو الاعتراض أو الطعن بأي صورة كانت في القرار الصادر عن مجلس العمداء بهذا الصدد.

توقيع الطالب: .... التاريخ: ٢ / ٦ / ٢٠١١

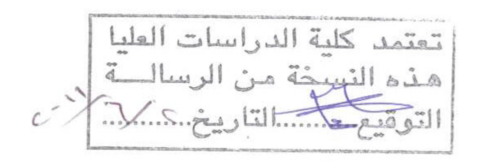
GOODNESS-OF-FIT TEST FOR THE WEIBULL DISTRIBUTION

By
Abd AL-Mjeed Alquliti

Supervisor
Dr. Ahmad Zghoul

Submitted in Partial Fulfillment of the Requirement for the
Master's Degree of science in Mathematics

Faculty of Graduate studies
The University of Jordan



May, 2011

This thesis (Goodness-of-Fit Test for The Weibul Distribution) was successfully defended and approved on 11/5/ 2011.

Examination Committee	**Signature**
Dr. Ahmad Zghoul, (Supervisor) Assoc. Prof. of Statistics, University of Jordan	....
Professor Dr. Adnan Awad, Member Prof. of Statistics, University of Jordan	....
Professor Dr. Mufid Azzam, Member Prof. of Statistics, University of Jordan	....
Dr. Helmi Kittaneh, Member Assoc. Prof. of Statistics, Hashemite University	....

ACKNOWLEDGEMENT

I would like to express my sincere appreciation to my supervisor Dr. Ahmad Zghoul, for his guidance, assistance and patience during this work, without his help this work would not have been finished.

My appreciation also goes to all other members of the Department of Mathematics, especially to Professor Rushdy Khalil, from whom I have learned a lot. Special thanks go to my parents for their love and constant encouragement.

Finally, my great appreciation goes to my wife for her support, understanding and patience while I was preparing this thesis.

TABLE OF CONTENTS

NOMENCLATURE

cdf Cumulative Distribution Function

pdf Probability Density Function

MME Method of Moment Estimation

MLE Maximum Likelihood Estimation

MIX Combination of both MLE and MME to estimate the scale and the shape parameters, respectively.

GPT Graphical Plotting Technique Estimation

EDF Empirical Distribution Function

A-D Anderson-Darling Test

C-M Cramer-von Mises Test

K-S Kolmogorov-Smirnov Test

L Liao-Shimokawa Test

S Mann, Scheure and Fertig Test

LIST OF FIGURES

LIST OF TABLES

GOODNESS-OF-FIT TEST FOR THE WEIBULL DISTRIBUTION

By
Abd AL-Mjeed Alquliti

Supervisor
Dr. Ahmad Zghoul

Abstract

In this thesis, we propose a new goodness-of-fit test for testing that a set of data is following a Weibull distribution. Both cases, known and unknown shape parameter, are considered. The maximum likelihood method of estimation is used to estimate the shape and the scale parameters when both parameters are unknown. Also, a mixture of the method of moments and the maximum likelihood method is employed to estimate both, the shape and the scale parameters.

Monte Carlo simulation is used to compute the critical values of the test statistic for different sample sizes and different choices of significance levels. Next, the performance of the test is studied by computing its power when testing the Weibull against some alternative distributions including; the normal, chi-square, Cauchy, Pareto, lognormal, gamma and other distributions.

Also, power comparisons are conducted between the proposed test and each of Kolmogorov-Smirnov, Cramer-von Mises, Anderson-Darling, and Liao-Shimokawa tests. The proposed test is proved to be competitive to the above tests when testing the Weibull against many alternatives.

Chapter one

Introduction

1.1 Background

An important problem in statistic is to obtain information about the form of the population from which a sample is drawn. Goodness of fit tests used to examine how well a sample of data agree with a given distribution.

Weibull distribution is widely used for fitting a wide range of types of data sets. In general, the objective of a statistical test is to evaluate a hypothesis concerning the values of one or more population parameters. All statistical tests of hypothesis work in the same way and consist of the same following elements, namely

- Null hypothesis, H_0
- Alternative hypothesis, H_a
- Test statistic
- Rejection region.

We decide between the null hypothesis, H_0 , and the alternative hypothesis, H_a.

The test statistic is computed from the sample data for deciding whether to reject or not to reject the null hypothesis H_0. The rejection region is the set of values of the test statistic for which the null hypothesis is rejected. Whereas the acceptance region is the set of values of the test statistic for which we fail to reject the null hypothesis. The critical of the test statistic is the boundary between the rejection region and the

acceptance region. If the value of the test statistic falls in the rejection region we conclude that the sample is not from the distribution assumed in H_0. Making a decision about the null hypothesis can be resulted in two types of errors, namely

- Type I error: which is the error committed by rejecting H_0 when H_0 is true. The probability of a Type I error is denoted by α.
- Type II error: which is the error committed by accepting H_0 when H_0 is false. The probability of a Type II error is denoted by β.

The quantity 1- β is called the power of the test. That is, the power of the test is the probability of rejecting the null hypothesis H_0 when it is false.

1.2 Objectives

Several goodness of fit tests for Weibull distribution have been introduced in the literature, but none of these tests is considered the best test in terms of the power for all sample sizes and all alternatives. For this reason there is a space to work in this area. This thesis aims at:

- Introducing a new goodness-of- fit test for the Weibull distribution that is compatible with already known tests. Two cases will be considered. In the first case, we assume that the shape parameter β is known, and in the second case we assume that β is unknown. However, in both cases the scale parameter θ is assumed to be unknown.
- Suggesting a new method to estimate β and θ simultaneously.
- Simulate critical values of the Proposed test at different significant levels.
- Simulate critical values for a set of known tests, at different significant levels. In fact critical value tables will be simulated for the Kolmogorov-Smirnov,

Cramer-von Mises, Anderson-Darling and the test proposed by Liao and Shimokawa.

- Computing the simulated power of the proposed test against a broad range of alternatives.
- Comparing the simulated power of the suggested test with those of well known tests.

CHAPTER TWO

LETERTURE REVIEW

2.1 Weibull Distribution

2.2 Estimation of Parameter

2. 3 Review of Goodness-of-Fit Test

2.4 Goodness of Fit Tests of Weibull Distribution

In this chapter, we will review the Weibull distribution and some of its properties. Also some methods of estimation for the Weibull distribution parameters will be discussed. Also, we will give a brief review of goodness-of-fit tests. Moreover,

We will review the goodness of fit tests of weibull distribution.

2.1 Weibull Distribution

The Weibull distribution was introduced by the Swedish physicist Waloddi Weibull in 1939 to represent the distribution of the breaking strength of materials. Since then the Weibull distribution plays an important role in reliability studies and life-testing applications in different fields. The Weibull distribution is a member of the family of the extreme-value distributions.

The probability density function (pdf) of the three-parameter Weibull is given by

$$f(x;\ \beta,\theta,\alpha) = \left(\frac{\beta}{\theta}\right)\left(\frac{x-\mu}{\theta}\right)^{\beta-1}\exp\left[-\left(\frac{x-\mu}{\theta}\right)^{\beta}\right],\quad -\infty < \mu \le x; \theta > 0, \beta > 0, \qquad (2.1)$$

and its cumulative distribution function (cdf) is given by

$$F(x;\beta,\theta,\alpha) = 1-\exp\left[-\left(\frac{x-\mu}{\theta}\right)^{\beta}\right],\quad \mu \le x\,. \qquad (2.2)$$

The parameter β controls the shape of the Weibull distribution. It makes the distribution very flexible. For example, when β = 1, the Weibull distribution becomes an exponential distribution and when β = 2, it is known as the Rayleigh distribution, and for values of β between 3 and 4 the distribution looks like a normal distribution. In

most lifetime estimation problems the shape parameter β is in the range 0.5 to 3.5. Figure(2.1) illustrates the flexibility of the density of the Weibull distribution for different values of the shape parameter β.

The scale parameter θ, describes the dispersion of the random variable about its mean, and is called, in reliability applications, the characteristic life. Figure(2.2) shows the Weibull distribution for different values of the scale parameter when the shape parameter is fixed. The location parameter or minimum life α, refers to the first failure that may occur.

The two- parameter Weibull is a special case of the three- parameter Weibull when $\mu = 0$.

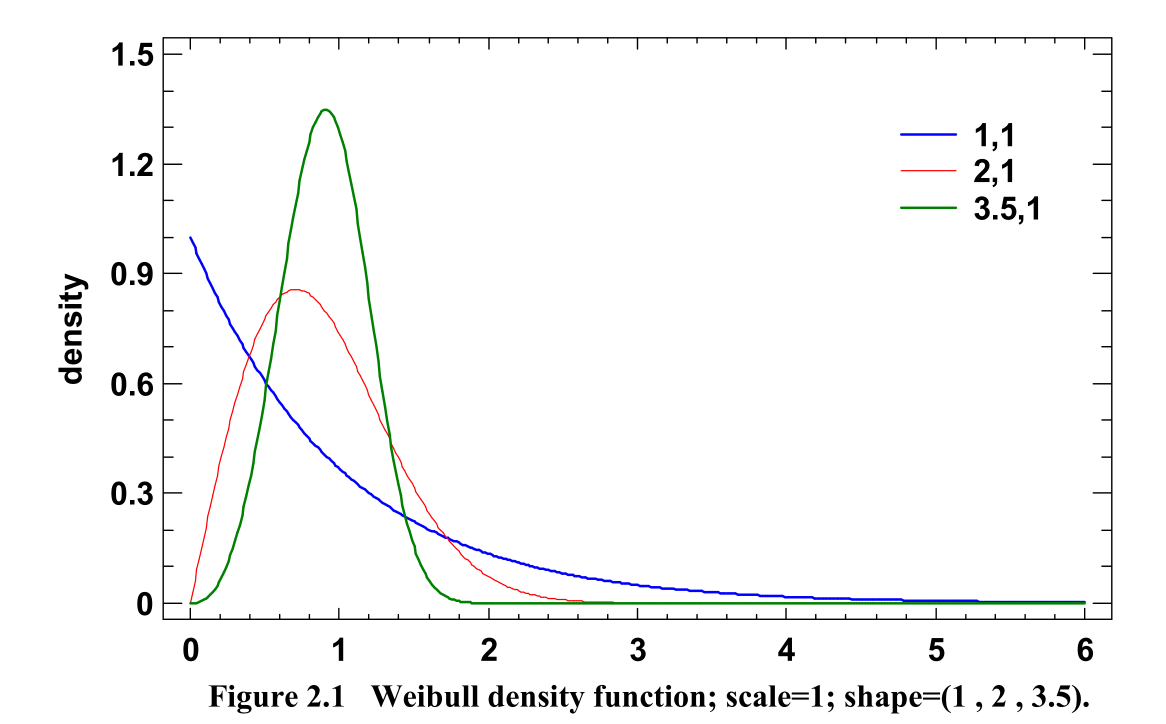


Figure 2.1 Weibull density function; scale=1; shape=(1 , 2 , 3.5).

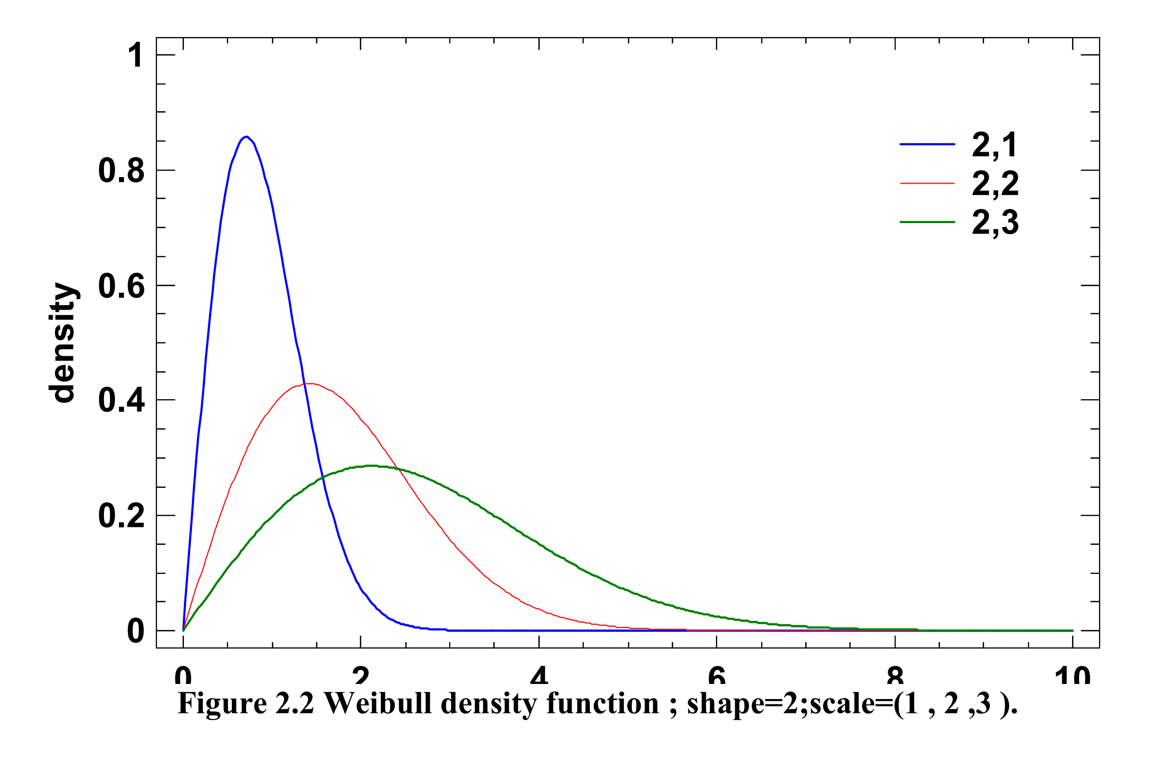


Figure 2.2 Weibull density function ; shape=2;scale=(1 , 2 ,3).

Weibull distribution has been used as a model in diverse disciplines to study many different issues. A main area of application for the Weibull distribution is lifetime data and reliability theory it is used to describe the life of roller bearings, electronic components, capacitors and dielectrics in accelerated tests . Also it is used in biological and medical applications. For more details, see for example Rinne (2009).

One of the good properties of the Weibull distribution is that it has different types of failure rate, that is; the Weibull distribution can be applied to different kinds of lifetime data. The failure rate is determined according to the shape parameter β. When $\beta < 1$ the failure rate decreases with time, when $\beta > 1$ the failure rate increases with time, and when $\beta = 1$ the failure rate is constant.

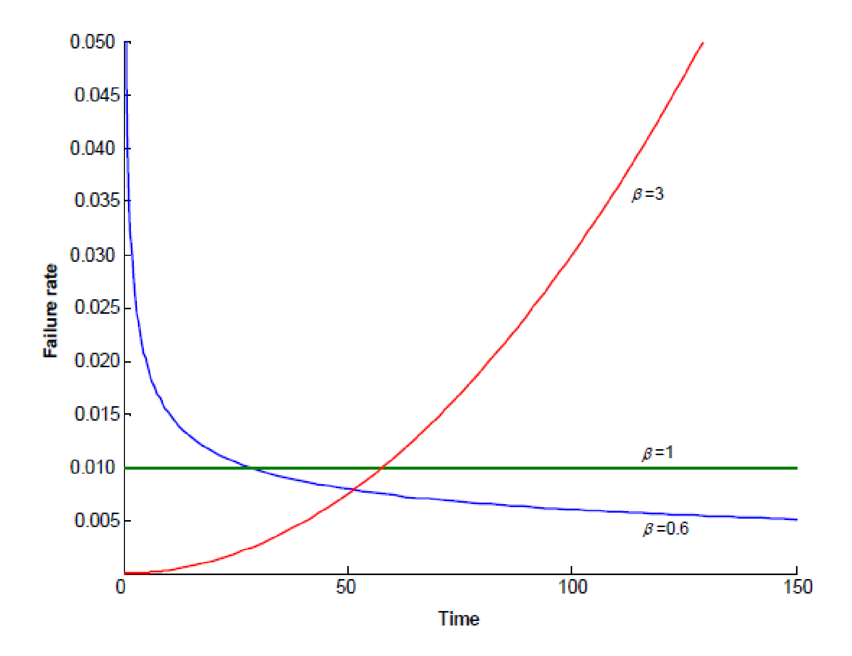


Figure 2.3 Failure rate functions of Weibull distribution with θ =100 and β = 0.6,1,3

From Figure 2.3, it is clear that the curves of the hazard rate function of the Weibull distribution are either increasing, constant or decreasing, which shows its flexibility to be used as a model for different types of data. Because of this property, Weibull distribution has been frequently used in estimating the reliability of products and systems.

The reliability of a system is the probability that, when operating under stated environmental conditions, the system will accomplish its intended function adequately in a specified interval of time, Kapur(1977).

The reliability (survival function) of the two- parameters Weibull distribution is

$$R(x) = 1 - F(x) = \exp\left[-\left(\frac{x}{\theta}\right)^{\beta}\right], \quad x > 0 \tag{2.5}$$

The failure rate (hazard function) of the two-parameters Weibull distribution is

$$h(x) = \frac{f(x)}{R(x)} = \frac{\frac{\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1} e^{-\left(\frac{x}{\theta}\right)^{\beta}}}{1-\left(1-e^{-\left(\frac{x}{\theta}\right)^{\beta}}\right)} = \left(\frac{\beta}{\theta}\right)\ \left(\frac{x}{\theta}\right)^{\beta-1}, \quad x > 0 \tag{2.6}$$

The central moments of the Weibull distribution are

$$E[(X-\mu)^r] = \theta^r \sum_{j=0}^{r} (-1)^j \binom{r}{j} \Gamma(\frac{r-j}{\beta}+1)\Gamma\left(\frac{j}{\beta}+1\right), \ r = 1,2, \ldots \tag{2.7}$$

The non-central moments of the Weibull distribution are given by

$$E[X^r] = \mu_r = \int_0^{\infty} x^r \left[\frac{\beta}{\theta}\right] \left(\frac{x}{\theta}\right)^{\beta-1} \ exp\left\{-\left(\frac{x}{\theta}\right)^{\beta}\right\} \ dx \ . \tag{2.8}$$

That is,

$$E(X^r) = \theta^r \Gamma\left[1+\frac{r}{\beta}\right]. \tag{2.9}$$

The mean and the variance of the two- parameters Weibull distribution are given by

$$E(X) = \theta\,\Gamma\left(1+\frac{1}{\beta}\right), \tag{2.10}$$

and

$$Var(X) = \theta^2\,\Gamma\left(1+\frac{2}{\beta}\right) - \left(\theta\,\Gamma\left(1+\frac{1}{\beta}\right)\right)^2. \tag{2.11}$$

Where $\Gamma(z) = \int_0^{\infty} t^{z-1}\ e^{-z}\ dt$ is the gamma function.

The Coefficient of variation is given by

$$v = \sqrt{\frac{\Gamma(2/\beta+1)}{\Gamma^2(1/\beta+1)}} - 1. \tag{2.12}$$

If X is a two-parameters Weibull random variable, then $Y = \log X$ has the Extreme value distribution (Log–Weibull distribution) with density (Rinne , 2009)

$$f(y) = \frac{1}{b^*} \exp\left\{\frac{1}{b^*(y - a^*)} - \exp\left[\frac{1}{b^*}\,(y - a^*)\right]\right\}, y \geq o, \tag{2.13}$$

Where a^* and b^* are the location and the scale parameters, respectively.

The relation between parameters of Weibull and Extreme value distributions are

$$a^* = \ln \theta \qquad and \quad \theta = \exp\,(a^*)$$

$$b^* = \frac{1}{\beta} \qquad and \quad \beta = \frac{1}{b^*},$$

where θ and β are the scale and the shape parameters for the Weibull.

Mean and variance for the Extreme value distribution are

$$\mathrm{E}(\log \mathrm{X}) = \mathrm{a}^* - \gamma\, \mathrm{b}^*, \quad \gamma \approx 0.577216\,, \tag{2.14}$$

and

$$\mathrm{Var}(\log \mathrm{X}) = \frac{\mathrm{b}^{*2}\pi^2}{6} \approx 1.644934\ \mathrm{b}^{*2}. \tag{2.15}$$

2.2 Estimation of Parameters

The parameters Estimation is considered to be an essential step when applying the goodness of fit test because in most cases the parameters are unknown and must be estimated from data. Many different methods can be applied to estimate the parameters of a Weibull distribution. Generally, these methods can be classified into two main classes; the graphical methods which include Weibull Probability Plot and Hazard Rate Plot, and the statistical methods which include the maximum likelihood estimation (MLE), minimum distance estimation (MDE), method of moment estimation (MME), Bayesian method, A comparison of different estimators was discussed by AL-Biadhani and Sinclair(1987).

The most common method of parameter estimation is the MLE method introduced by Fisher, in 1920. This method selects the values that maximize the likelihood function of the observed sample as estimators for the parameters. Where the likelihood function is the joint density function of the sample.

Maximum likelihood estimators (MLE) have several desirable statistical properties. Some of these properties are summarized here:

1. For most common distributions, the MLE is unique; there are no alternative values that maximize the likelihood function.
2. MLEs are invariant, that is if $\hat{\theta}$ is the MLE of θ then $h(\hat{\theta})$ is the MLE for $h(\theta)$ where $h(\theta)$ is a function of θ.
3. MLEs are asymptotically normally distributed.
4. MLEs are strongly consistent, that is , $\lim_{n\to\infty}\mathrm{Var}(\hat{\theta}) = 0$.

Let $X_1, X_2, \dots, X_n$ be a set of independent random variables from a distribution with PDF $f(x_i;\ \theta)$, where $\underline{\theta}$ is a vector of parameters. The likelihood function is given by

$$l(x_1, x_2, \dots, x_n\ ; \theta) = \prod_{i=1}^{n} f(x_i;\ \theta)\,.$$

For the two-parameters Weibull distribution, the likelihood function is

$$l(x_1, x_2, \dots, x_n\ ; \beta, \theta) = \prod_{i=1}^{n} f(x_i\ ;\ \beta, \theta).$$

$$= \left(\frac{\beta}{\theta}\right)^n \prod_{i=1}^{n} (x_i)^{\beta-1} \exp\left[-\sum_{i=1}^{n} \left(\frac{x_i}{\theta}\right)^{\beta}\right]. \qquad (2.16)$$

We note from(2.16) that the likelihood function increases without bounds to infinity when $\beta < 1$. So, for the MLE to be exist β should be greater than or equal to one.

By taking the log –likelihood function we get

$$\log l = n \log \beta - n \log \theta + (\beta - 1) \sum_{i}^{n} \mathrm{Log}\left(\frac{x_i}{\theta}\right) - \sum_{i=1}^{n} \left(\frac{x_i}{\theta}\right)^{\beta}.$$

Taking the partial derivatives of the last equation, with respect to β and θ, respectively, and equating these derivatives to zero, we get

$$\frac{\partial(\log l)}{\partial \beta} = \frac{n}{\beta} + \sum_{i=0}^{n} \log(x_i) - \theta^{-\beta} \sum_{i=1}^{n} \left[(x_i)^{\beta} \log\left(\frac{x_i}{\theta}\right)\right] = 0,$$

and

$$\frac{\partial(\log l)}{\partial \theta} = -\frac{\beta n}{\theta} + \frac{\beta}{\theta^{\beta+1}} \sum_{i}^{n} {x_i}^{\beta} = 0.$$

Then $\hat{\beta}$ is computed by solving the equation:

$$\frac{n}{\beta} + \sum_{i=0}^{n} \log(x_i) - \frac{n}{\sum_i^n {x_i}^{\beta}} \sum_i^n {x_i}^{\beta} \log(x_i) = 0 \qquad (2.17)$$

When the shape parameter is estimated then the scale parameter, $\hat{\theta}$ could be estimated by

$$\hat{\theta} = \left[\frac{1}{n}\sum_{i}^{n} x_i^{\hat{\beta}}\right]^{\frac{1}{\hat{\beta}}}. \quad (2.18)$$

Since these equations are impossible to be solved explicitly , we need to use numerical methods, such as Newton-Raphson method, to solve for the unknown parameters.

Menon (1963) proposed another method of estimation. These estimators are moment estimators of the parameters of the Log–Weibull distribution ,

Mean and variance for the Extreme value distribution are

$$E(X) = a^* - \gamma\, b^*, \quad where \quad \gamma \approx 0.577216,$$

and

$$Var(X) = \frac{{b^*}^2\pi^2}{6} \approx 1.644934\, {b^*}^2$$

See, for example, Rinne (2009)

Thus, the moment estimators of b^* (given by Menon ,1963) is

$$\widehat{b^*} = \frac{\sqrt{6}}{\pi} S_X \approx 0.77969\, S_X , \quad (2.19)$$

Thus, the shape parameter β of the Weibull distribution estimated as $\hat{\beta} = \frac{1}{\widehat{b^*}}$

Thoman(1969) compared the MLEs and Menon estimators for complete samples of different sizes. The biases for the two estimators of the shape parameter are almost equal. Cohen(1965), Bain (1972) , Engelhardt and Bain (1973; 1974), Engelhardt (1975) and Mann and Fertig(1975) all of them discuss the procedure of the Maximum likelihood estimation for the two-parameters Weibull distribution. However, finding MLEs for the three parameter Weibull distribution is difficult from the computational point of view. Numerous studies addressed the problem of solving the maximum

likelihood equations for the three-parameter Weibull distribution numerically. Harter and Moor(1965) , Gallagher(1990), Lemon(1975), Zanakis(1977), Cohen and Whitten(1988), all of them discussed the computational problem for the three-parameter Weibull distribution. In fact, when $\beta < 1$, the likelihood function increases without bound as the location parameter α tends to $X_{(1)}$, in which case the MLEs for the shape and scale parameters do not exist. Harter and Moor(1965) brought to attention that, when the location estimate is larger than $X_{(1)}$, there is a problem because $ln(X_{(1)} - \hat{\alpha})$ is not defined. They suggested censoring the sample less than or equal to α. Cohen and Whitten (1988) recommended using other methods of estimation when $\beta > 2.2$. Engelhardt and Bain (1974) introduced what is called the good linear unbiased estimator (GLUE) for the extreme value distribution parameters. However, for three-parameter Weibull this method requires the shape or the location parameter to be known. Cohen and Whitten (1988) used the Method of Moment estimation (MME) for the three–parameter Weibull distribution. This method is computationally easier to apply than the MLE and it can be applied for $\beta < 1$. However, Cohen (1988) noted that MME have larger variances than MLE.

Another important method of estimating parameters is the Minimum Distance Estimation (MDE). The values of the parameters that minimizes the distance between the cumulative distribution function (CDF) and the empirical distribution function (EDF)are considered estimates of these parameters. Moore and Gallagher (1990) noted that for the three-parameter Weibull , the two methods of estimation together ; MLE for the scale and shape parameters and MDE for the location parameter is the best method among the several alternative methods including the MLE.

2.3 Review of Goodness -of-fit Tests

Goodness-of- fit test is used to check whether a given sample of data comes from a hypothetical distribution. That is, given a random sample $X_1, \dots, X_n$ from a population with some distribution function F(x) , we want to test if F(x) is some known distribution function $F_0(x)$;

$$H_0 : F(x) = F_0(x) \qquad \text{vs} \qquad H_a = F(x) \neq F_0(x).$$

There are several goodness-of- fit tests and none of these tests has the best power against all alternatives. As mentioned earlier, the power of the test is the probability of rejecting the null hypothesis when the sample is taken from a distribution other than $F_0(x)$. The higher the power of a test , the lower the chance of accepting a distribution when it is false.

If the null distribution and all its parameters are completely specified, then we have a simple hypothesis, where if one or more of the distribution parameters are unknown, then the hypothesis is composite.

The general procedure for any goodness-of- fit test can be summarized in the following steps:

- Choose a hypothesized distribution.
- Estimate the parameters for the distribution if any.
- Calculate the test statistic.
- Calculate the probability of rejection of H_0.

There are several classical goodness-of-fit tests, some of these tests are:

- Chi-Square
- Kolmogorov-Smirnov
- Anderson-Darling, and
- Cramer von Mises goodness of fit test.

2.3.1 Chi-Squared Test

The Chi-Squared test, is the oldest known goodness-of-fit test, introduced by Karl Pearson in 1900. The Chi-Square test statistic is defined as

$$\chi^2 = \sum_{i=1}^{k} \frac{(o_i - e_i)^2}{e_i}, \qquad (2.20)$$

where ,

o_i is the observed frequency in class i, e_i is the expected frequencies in class i, and k is the number of classes.

The advantages of the Chi-Square test is that it can be easily computed and it can be applied to both continuous and discrete variables. Also the asymptotic distribution of the test statistic is Chi-Square distribution with (k-1-s) degrees of freedom ($\chi^2_{(k-1-s)}$), where s is the number of the estimated parameters under the null hypothesis. However, the Chi-Square test is usually less powerful than other tests, also it is not recommended for small sample size (less than 25).

The null hypothesis is rejected at level α if the value of the statistic $\chi^2 > \chi^2_{\alpha(k-1-s)}$.

2.3.2 Empirical Distribution Function Tests

Given a random sample $X_1, \dots, X_n$ from a population with distribution function $F(x)$, let $X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(n)}$ be the corresponding order statistics. The empirical distribution function (EDF) of the sample is a step function, which estimates the population distribution function. Specifically, the empirical distribution function of $X_1, \dots, X_n$ is defined by

$$F_n(x) = \begin{cases} 0, & x < X_{(1)} \\ \frac{i}{n}, & x_{(i)} \leq x < X_{(i+1)}, \; i = 1, \dots, n-1 \\ 1, & X_{(n)} \leq x \end{cases} \quad (2.21)$$

So for any real number x, $F_n(x)$ records the proportion of observations less than or equal to x. We can use $F_n(x)$ to estimate F(x) and it is in fact a consistent estimator of F(x), that is as $n \longrightarrow \infty$, $|F_n(x) - F(x)|$ goes to zero with probability one, see for example D' Agostino (1986).

2.3.3 Empirical distribution function statistics

A statistic measuring the distance between $F_n(x)$ and a hypothetical distribution with F(x) will be called EDF statistic. when the hypothesized distribution F(x) is correct, we can observe that the two distributions are close to each other. Figure 3 illustrates the relation between these two functions; $F_n(x)$ and F(x).

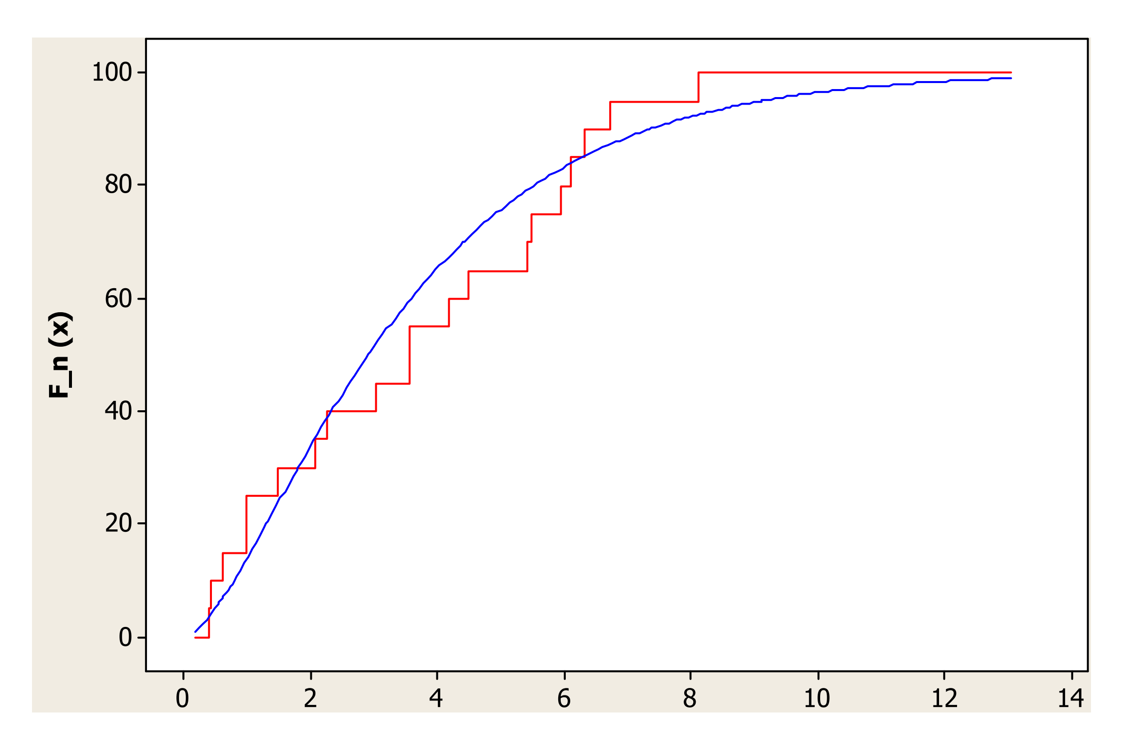


Figure 2.4 Empirical distribution function for a sample of size 20 from Weibull distribution.

when the hypothesized distribution is completely specified (all parameters are known), the EDF statistic does not depend on the hypothesized distribution. Therefore, we can use only one table of critical values for all distributions for a given test statistic. But when the hypothesized distribution is composite, which is usually the case, then some or all the parameters must be estimated and the EDF statistic depends on the hypothesized distribution, the sample size, the estimated parameter(s), and the method of estimation. However, in the cases where the location and/ or scale parameters are unknown and estimated with invariant estimators such as MLE, or minimum distance estimators, test statistics will not depend on the true values of location and scale parameters , but will only depend on the CDF and the sample size (David and Johanson,1948). When a shape parameter is unknown and must be estimated, then the test statistic will depend on the true value of the parameter, and different critical values tables must be constructed for each value of the shape parameter.

There are two main classes of EDF statistics; namely, the supremum class and the quadratic class. The statistics based on the largest differences between $F_n(x)$ and F(x) belong to the supremum class. The most important tests in this class are Kolmogorov-Smirnov test (1933) and Kuiper test (1960).

The statistics based on integrating the squared differences between $F_n(x)$ and F(x) belong to the quadratic class. The most important quadratic tests are the Cramer-von Mises statistic (1928) and the Anderson-Darling statistic (1954). In general, the EDF tests are more powerful than the Chi-Square test especially for small samples, D'Agostino and Stephens (1986).

2.3.4 Kolmogorov-Smirnov Test

The first known EDF test statistic is the Kolmogorov-Smirnov test (Kolmogorov 1933; Smirnov 1939). This test is one of the simplest goodness of fit tests. It is based on the supremum measure of the difference between the empirical distribution function $F_n(x)$ and the hypothesized distribution function F(x). Kolmogorov (1933) introduced the test statistic as follow

$$D = \sup_x |F_n(x) - F_0(x)|,$$

or in an equivalent form as

$$D = max(D^+ , D^-), \tag{2.22}$$

where

$$D^+ = \max_x \left(\frac{i}{n} - F(x)\right), \text{ and } D^- = \max_x \left(F(x) - \frac{i-1}{n}\right).$$

The distribution under H_0 will be rejected if the value of the statistic is greater than or equal to a corresponding critical value.The critical values when the parameters are unknown for the weibull distribution are given in Littel et al. (1979).

2.3.5 KuiperTest

Kuiper (1960) introduced a modification of Kolmogorov-Smirnov test D. The modified test is of the form

$$V = D^{+} + D^{-} . \tag{2.23}$$

where D^{+} and D^{-} are as defined above.

2.3.6 Cramer-von Mises Test

The most-well known EDF goodness-of- fit test belong to the quadratic type test is the Cramer-von Mises test defined by

$$Q = n\int_{-\infty}^{\infty}\left(F_n(x) - F(x)\right)^2 w(\mathrm{x})\, \mathrm{d}F(x), \tag{2.24}$$

where w(x) is a suitable weight function.

When $w(\mathrm{x}) = 1$, the test statistic is reduced to W^2

$$W^2 = n\int_{-\infty}^{\infty}\left(F_n(x) - F(x)\right)^2 \mathrm{d}F(x). \tag{2.25}$$

2.3.7 Anderson-Darling Test

Another important test statistic of quadratic type is the test proposed by Anderson and Darling (1954), denoted by A^2. The A^2 test is obtained when the weight function w(x) has the form $\mathrm{w(x)} = \frac{1}{\mathrm{F(x)(1-F(x))}}$. That is,

$$A^2 = n\int_{-\infty}^{\infty}\left(F_n(x) - F(x)\right)^2 [\mathrm{F(x)(1-F(x))}]^{-1}\, \mathrm{d}F(x). \tag{2.26}$$

As in the K-S test, the null hypothesis is rejected if the value of the statistic is large. The critical values for A^2 test where $F_0(x)$ is the Weibull or the Type I extreme value distribution can be obtained using Monte Carlo simulations at different significance levels and sample sizes, (Stephens, 1977; Littel *et al.*, 1979).

Many authors, for example Stephens(1974), Lawless (1982), and Liao and Shimokawa (1999), found that the Anderson-Darling and the Cramer-von Mises test statistics are more powerful than the Kolmogorov-Smirnov test for all considered alternatives.

2.3.8 Watson Test

Watson (1961) proposed a test statistic as a modification of the Cramer-von Mises statistic W^2. The test statistic is defined as

$$U^2 = n\int_{-\infty}^{\infty}\left\{F_n(x) - F(x) - \int_{-\infty}^{\infty}[F_n(x) - F(x)]\, \mathrm{d}F(x)\right\}^2 \mathrm{d}F(x). \qquad (2.27)$$

For calculating the Cramer-von Mises W^2, Anderson-darling A^2, and watson U^2 tests, applying the probability integral transformation, $Z_i = F\left(x_{(i)}\right),\ i = 1,2,\dots,n$, where F(x) is the hypothesized CDF, the computational forms for $\mathrm{W}^2, \mathrm{A}^2$ and U^2 tests can be obtained. These are given in D'Agostino and Stephens (1986) ⍰^{2} , W^2, U^2 as follows:

$$\mathrm{W}^2 = \frac{1}{12n} + \sum_{i=1}^{n}\left(Z_i - \frac{(2i-1)}{2n}\right)^2, \qquad (2.28)$$

$$\mathrm{A}^2 = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-2)[logZ_i + log(1 - Z_{n+1-i})], \qquad (2.29)$$

and

$$U^2 = \frac{1}{12n} + \sum_{i=1}^{n}\left(Z_i - \frac{(2i-1)}{2n}\right)^2, + \sum_{\mathrm{i}=1}^{\mathrm{n}}\left[\frac{\mathrm{Z_i}}{\mathrm{n}} - \frac{1}{2}\right]^2. \qquad (2.30)$$

To illustration the goodness-of- fit test procedure, a numerical example will be discussed.

Consider the following sample of data, n = 10 which is the number of cycles to failure of springs taken from Stephens and Lockhart (1994)

225 171 198 189 189 135 162 135 117 162

Our interest is to test

$H_o: F(x) = F_0(x),$ versus $H_a: F(x) \neq F_0(x),$

where $F_0(x)$ is Weibull cumulative function.

The MLE of θ and β are:

$$\hat{\theta} = 181.40 \quad \text{and} \quad \hat{\beta} = 5.97$$

Kolmogorov-Smirnov(K-S) test statistic D is

$$D = max(D^+, D^-) = .1427$$

The corresponding critical value of the K-S test statistics D from the table (3.13) for n=10 and level of significance $\alpha = 0.05$ is $D_{.05} = 0.260$

Since the test statistic D is obtained as 0.143, which is less than the critical value of 0.260 at 0.05 significance level. We do not reject The null hypothesis that the sample come from the Weibull distribution.

2.4 Goodness-of-fit Test For the Weibull Distribution

Goodness -of-fit test for the Weibull distribution are usually performed on the log-data rather than on the raw data, in this case shape and scale parameters are converted to scale and location parameters Tiku and Singh (1981), Wozniak and Li (1990). Therefore, goodness of fit test for the extreme value distribution is equivalent to the goodness-of- fit test for the Weibull distribution, Tiku and Singh (1981).

Several papers addressed the goodness-of-fit test problem for the two and three - parameters Weibull distribution when the parameters are unknown. Lawless (2003) summarized the goodness-of-fit tests for the Weibull and extreme value distributions.
Mann et al.(1973) developed a test statistic S, for testing the two-parameters Weibull distribution with unknown parameters based on normalized spacing.
The S statistic is given by

$$S = \frac{\sum_{i=\frac{n}{2}+1}^{n-1} G_i}{\sum_{i=1}^{n-1} G_i},$$

Where G_i, is given by

$$G_i = \frac{X_{(i+1)} - X_{(i)}}{\mu(i+1:n) - \mu(i:n)}.$$

Where $X_{(i)}, i = 1, \dots, n$ are the order statistics of a sample of size n.
Their test statistic is the ratio of a linear combination of the sum of order statistics divided by the expected values of a normalized sum of order statistics from a censored sample. The advantage of the S statistic is that it is relatively easy to calculate and the parameters of the Weibull distribution do not need to be known nor estimated.
Their power study showed that their test has a higher power, compared to known tests such as K-S, C-M and A-D tests for all alternatives under consideration such as normal, log normal, logistic and $\chi^2(4)$ distributions.

This test was modified by Tiku and Singh (1981), the modified test is given by

$$Z^* = \frac{2\sum_{i=1}^{n-2}(n-1-i)G_i}{(n-2)\sum_{i=1}^{n-1}G_i}$$

Where G_i, is given by

$$G_i = \frac{X_{(i+1)} - X_{(i)}}{\mu(i+1{:}n) - \mu(i{:}n)}.$$

Where $X_{(i)}, i = 1, \dots, n$ are the order statistics of a sample of size n. The modified test proved higher power then Mann el al.(1973) test for a wide range of alternatives.

Stephens (1977) produced tables of critical values for the Anderson-Darling and for the Cramer von Mises tests for various significance levels.

Littell et al.(1979) modified each of K-S, A-D, and C-M tests and applied these tests for the two-parameter Weibull distribution with unknown parameters. They build the critical values table for sample of sizes 10(5)40 and significance levels α= 0.01, 0.05, 0.10, 0.15, 0.20. In their power study the modified A-D and C-M test
is the most powerful goodness-of-fit test among the corresponding K-S, C-M, A-D and S tests when testing against alternatives such as $\chi^2(1)$, $\chi^2(4)$, log-normal, Cauchy, logistic, double Exponential and normal distributions.

Woodruff et al. (1983) modified K-S test for the Weibull distribution with unknown location and scale parameters. Maximum likelihood estimation was used for both the location and the scale parameters. Bush et al.(1983) obtained critical values for their modified A-D and C-M statistics for the three-parameter Weibull distribution when the scale and location parameters are unknown and the shape parameter is known. In their power study they found that the A-D and C-M tests are more powerful than the K-S and Chi-Square tests for a broad range of alternatives. Coles (1989) proposed a test statistic based on stabilized probability plot for testing goodness of fit for two-parameter Weibull distribution with unknown parameters.

Liao and Shimokawa (1999) proposed a test statistic L, based on a combination of the three classical tests; Kolmogorov - Smirnov, Cramer-von Mises, and Anderson – Darling for testing the two- parameter Weibull distribution and extreme–value distribution.

The L test statistic has the form

$$L = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \frac{\max\left[\frac{i}{n} - F_0(x_i,\hat{\theta}), F_0(x_i,\hat{\theta}) - \frac{i-1}{n}\right]}{\sqrt{F_0(x_i,\hat{\theta})[1 - F_0(x_i,\hat{\theta})]}} . \qquad (2.31)$$

Maximum likelihood estimator (MLE) and graphical plotting technique (GPT) used to estimate the parameters. In their Monte Carlo simulation, they used 1,000,000 runs to build the critical values tables for sample of sizes 3(1)20,25(5)50,60(10)100 and significance levels α= 0.025, 0.05, 0.10, 0.15, 0.20, 0.25. From their power study, they concluded that the test statistic L with the (GPT) is the most powerful test compared to the K-S, C-M, A-D and S tests when testing against alternatives such as $\chi^2(1)$, $\chi^2(4)$, log-normal, Cauchy, logistic and normal distributions.

Shapiro and Brain (1987) proposed test statistic based on the comparison of two different estimators of the scale parameters of the log- Weibull distribution. In their study they used two different scale estimators which are linear unbiased estimators; the two estimator are:

$$S^2 = \sum_{i=1}^{n} (X_i - \bar{X})^2, \qquad (2.32)$$

and, the second estimator is D'Agostino (1971) estimator, which can be written as

$$b = \frac{1}{n}\left(.6079 \sum_{i=1}^{n} W_{n+i} \ X_{(i)} - .2570 \sum_{i=1}^{n} W_i \ X_{(i)} \right), \qquad (2.33)$$

where

$$W_i = \ln\left(\frac{n+1}{n+1-i}\right), i = 1, 2, \dots, n-1$$

$$W_n = n - \sum_{i=1}^{n-1} W_i ,$$

$$W_{n+i} = W_i\ (1 + \ln W_i\ ) - 1\ \ , i = 1, 2, \dots , n - 1$$

$$W_{2n} = .4228n - \sum_{i=1}^{n-1} W_{n+i}.$$

Their proposed statistic can be written as

$$W = \frac{nb^2}{S^2}\ , \qquad (2.34)$$

They used 40,000 runs to build the critical values table for sample of sizes 3(1)10(2)20(5)50(10)100 and different significance levels.

In their power study they used 4000 run to found that the W test is more powerful than the S test for a broad range of alternatives such as: $\chi^2(1)$, $\chi^2(3)$, $\chi^2(4)$,Pareto(0,1) and Normal(1,.2). Later, Ozturk and Korukoglu(1988) proposed a test statistic which is a modification of the Shapiro and Brain test. They used D'Agostino estimator denoted by b and the second estimator is the probability weighted estimator given by

$$\hat{\sigma} = \sum_{i=1}^{n} (2i - n - 1)X_i \Big/ [.6931\ n(n - 1)].$$

Their proposed test has the form

$$W^* = \frac{b}{\hat{\sigma}}. \qquad (2.35)$$

They concluded that their test statistic is more powerful than Shapiro and Brain test, also they found that their test was computationally easier to apply.

CHPTER THREE

METHODOLOGY

3.1 The test statistic

3.2 Monte Carlo simulation

3.3 Computation of the Critical Values

3.4 Power Study

3.5 Comparative Power Study

3.6 Conclusions and Recommendations

In this chapter, the proposed test statistic T for testing the two- parameter Weibull distribution, based on the ratio of two estimators of the shape parameter β will be discussed. Because the exact distribution of the proposed test is not available, which is the case in most goodness-of-fit tests. Monte Carlo simulation with 20,000 runs will be used to obtain the proposed test quantiles and critical values tables.

Then, Monte Carlo simulations with 10,000 repetitions will be used to compute the power for the proposed test against a wide range of alternative distributions.

Also, power comparisons are conducted between the proposed test T and Anderson-Darling test (A^2), Cramer von Mises test (W^2), Kolmogorov-Smirnov test (K-S) and Liao- Shimokawa test (L).

3.1 The Test Statistic

Let $X_1, \ldots, X_n$ be iid random variables with density $f(x, \underline{\theta})$. We test the hypothesis that the sample is coming from a two parameters Weibull distribution with density function

$$f(x;\, \beta, \theta) = \left(\frac{\beta}{\theta}\right)\left(\frac{x}{\theta}\right)^{\beta-1} exp\left[-\left(\frac{x}{\theta}\right)^{\beta}\right], \quad x > 0 \tag{3.1}$$

We will develop a test statistic that is based on the ratio of two estimators of the shape parameter β. In fact, this idea is common in goodness of fit literature; for example, the prominent Shapiro-Wilk test of normality proposed by Shapiro and Wilk (1965) is simply a ratio of two estimators of the variance. Also, Wilk(1972) proposed a test for exponentiality based on the ratio of the sample mean and the sample standard deviation as each of these two statistics is an estimator of the exponential scale parameter. Moreover, Shapiro and Brain (1987) proposed a test statistic W for the

Weibull distribution based on the comparison of two different estimators of the scale parameters of the log- Weibull variate. Also, Ozturk and Korukoglu(1988) proposed a test for the Weibull which is a modification of the Shapiro and Brain test.

It can be shown that if X is a random variable with Weibull density as given in(3.1), then

$$E\left[\mathrm{Log}\left(\frac{X}{\theta}\right)\right] = \frac{-\gamma}{\beta}, \qquad (3.2)$$

and

$$\mathrm{Var}\left[\mathrm{Log}\left(\frac{X}{\theta}\right)\right] = \frac{\pi^2}{6\beta^2}. \qquad (3.3)$$

Where γ is the Euler constant given by:

$$\gamma = -\Gamma'(1) = -\int_0^\infty e^{-x}\ln(x)dx \approx 0.5772156649. \qquad (3.4)$$

See for example, Johnson et al.(1995)

Let $Y_i = \mathrm{Log}\left(\frac{X_i}{\theta}\right)$, $i = 1,2,\dots,n$. Then the empirical counter part of

$E\left[\mathrm{Log}\left(\frac{X}{\theta}\right)\right]$ is the statistic $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$ and the empirical counter part of

$\mathrm{Var}\left[\mathrm{Log}\left(\frac{X}{\theta}\right)\right]$ is the statistic $S_Y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \bar{Y})^2$. Obviously, when θ is known

$\frac{-\bar{Y}}{\gamma}$ is an unbiased estimator for $\frac{1}{\beta}$ and $\frac{6S_Y^2}{\pi^2}$ is an unbiased estimator for $\frac{1}{\beta^2}$.

These estimators are moment estimators. Our test statistic will be based on the ratio of

$\bar{Y}^2$ to S_Y^2. Actually, one can propose any of the following tests:

- Class I $\quad \frac{\bar{Y}}{S_Y}, \left(\frac{\bar{Y}}{S_Y} - E\left[\frac{\bar{Y}}{S_Y}\right]\right), \left(\frac{\pi}{\gamma\sqrt{6}}\frac{\bar{Y}}{S_Y}\right), \left(\frac{\pi}{\gamma\sqrt{6}}\frac{\bar{Y}}{S_Y} - 1\right).$
- Class II $\quad \left|\frac{\bar{Y}}{S_Y}\right|, \left|\frac{\bar{Y}}{S_Y} - E\left[\frac{\bar{Y}}{S_Y}\right]\right|, \left(\frac{\pi}{\gamma\sqrt{6}}\frac{|\bar{Y}|}{S_Y}\right), \left(\frac{\pi}{\gamma\sqrt{6}}\frac{|\bar{Y}|}{S_Y} - 1\right).$
- Class III $\quad \frac{\bar{Y}^2}{S_Y^2}, \left(\frac{\bar{Y}^2}{S_Y^2} - E\left[\frac{\bar{Y}^2}{S_Y^2}\right]\right), \left(\frac{\pi^2}{6\gamma^2}\frac{\bar{Y}^2}{S_Y^2} - 1\right).$

- Class IV $\left|\frac{\bar{Y}}{S_Y{}^2}^2 - E\left[\frac{\bar{Y}}{S_Y{}^2}^2\right]\right|, \left|\frac{\pi^2}{6\gamma^2}\frac{\bar{Y}}{S_Y{}^2}^2 - 1\right|.$

The test statistics in classes I and III produce two sided tests, whereas tests in classes II and IV are one sided tests. All tests listed above are invariant for the choice of the shape parameter β. One may propose tests based on differences of estimators, but those tests will depend on β. Therefore, we will choose our test from tests based on ratio of estimators of β rather than tests based on differences of such estimators.

An initial power study suggests that the test $T = \left|\frac{\pi^2}{6\gamma^2}\frac{\bar{Y}}{S_Y{}^2}^2 - 1\right|$ may have the highest power against many alternatives including the lognormal(0,1), gamma(2,1), folded Cauchy(0,1) and folded normal compared to other tests listed in class I to class IV above. Therefore, we adopt this test or equivalently its square to test for a two parameters Weibull distribution.

By laws of large numbers (Petrov 1995), we have $\frac{\pi^2}{6\gamma^2}\frac{\bar{Y}}{S_Y{}^2}^2 \underset{D}{\rightarrow} 1 \quad as \text{ n} \rightarrow \infty.$

Thus, T is nothing but distance between the normalized and squared reciprocal of the coefficient of variation of the transformed sample $Y_1, \ldots, Y_n$ and its asymptotic mean.

We mentioned above that the test T is invariant of β, to clarify this we have

$$\frac{\bar{Y}}{S_Y{}^2}^2 = \frac{\left(\frac{1}{n}\sum_{i=1}^n \log\left(\frac{x_i}{\theta}\right)\right)^2}{\frac{1}{n-1}\left[\frac{1}{n}\sum_{i=1}^n\left[\log\left(\frac{x_i}{\theta}\right)\right]^2 - n\left(\frac{1}{n}\sum_{i=1}^n \log\left(\frac{x_i}{\theta}\right)\right)^2\right]}$$

$$= \frac{\frac{1}{\beta^2}\left(\frac{1}{n}\sum_{i=1}^n \log\left(\frac{x_i}{\theta}\right)^\beta\right)^2}{\frac{1}{\beta^2}\left[\frac{1}{n-1}\left\{\frac{1}{n}\sum_{i=1}^n\left[\log\left(\frac{x_i}{\theta}\right)^\beta\right]^2 - n\left(\frac{1}{n}\sum_{i=1}^n \log\left(\frac{x_i}{\theta}\right)^\beta\right)^2\right\}\right]}$$

$$= \frac{\bar{W}}{S_W{}^2}^2,$$

where $W_i = \log\left(\frac{X_i}{\theta}\right)^{\beta}$.

In case θ and β are unknown, they are replaced by their estimates and the

transformation $W_i = \log\left(\frac{X_i}{\hat{\theta}}\right)^{\hat{\beta}}$ is applied.

From the above, we conclude that

$$T = \left(\frac{\pi^2}{6\gamma^2}\frac{\bar{Y}}{S_Y{}^2}^2 - 1\right)^2 = \left(\frac{\pi^2}{6\gamma^2}\frac{\bar{W}}{S_W{}^2}^2 - 1\right)^2. \quad (3.5)$$

If the parameters β and θ are unknown they should be estimated from the sample. We noticed in our discussion in Chapter Two of this thesis that the MLE for the Weibull parameters involve some problems, for example, MLE does not exist for $\beta < 1$. In this thesis we propose estimating the shape parameter using the MME and the scale parameter by the MLE. In fact, we estimate β and θ by:

$$\hat{\beta} = \frac{\pi}{\sqrt{6}\, S_Y} \quad \text{and} \quad \hat{\theta} = \left[\frac{1}{n}\sum_i^n x_i{}^{\hat{\beta}}\right]^{\frac{1}{\hat{\beta}}},$$

and we will here after refer to these estimators as a mixture estimators, as they are a mixture of MME and MLE methods, and will be abbreviated by MIX.

3.2 Monte Carlo Simulation

Simulation is a numerical technique for conducting experiments on a digital computer. These techniques involve certain types of mathematical and logical models that describe the behavior of business or economic system, Naylor (1966).

Monte Carlo Simulation is widely used to solve problems in statistics that are not analytically tractable. A Monte Carlo simulation uses a large number of random numbers to solve problems where the passage of time plays no important role, Kelton(1991). More specifically , Monte Carlo simulation generate a random sample from a particular distribution and uses the sample to evaluate some measures of interest. This process is repeated for several samples. The Monte Carlo method can be used for difference problems like solution of stochastic problems, integral and differential equations and sampling of random varieties. For more details, see Law and Kelton(1991).

The Monte Carlo process is used in this thesis to generate the critical value tables for the proposed goodness-of-fit test, also to evaluate the power of the new test. Most of the studies until today show that 10,000 to 20,000 repetition provide consistent results. So, in this thesis we will use 20,000 repetitions for critical values computations and 10,000 for power evaluations.

3.3 Computation of the Critical Values

To obtain the critical values for the proposed goodness of fit test for two-parameter Weibull distribution , we use Mathematica version 7 to simulate data from the designated distribution and compute the value of the test statistic for each simulated sample. For each sample of size n =5(5)100 and 200 using 20,000 repetitions, the following set of quantiles of the test are computed:

$\{0.5, 0.75, 0.90, 0.95, 0.975, 0.99, 0.995\}$

These quantiles will be computed for the following cases:

Case1: The shape parameter β is known, while the scale parameter θ is unknown and estimated by the MLE method.

Case2: The shape and the scale parameters are both unknown and are estimated by the MLE.

Case3: The shape and the scale parameters are both unknown and are estimated by a combination of MME and MLE.

The Monte Carlo procedure for obtaining the critical values can be described in the following steps:

1. Generate a sample of size n from Weibull (θ,β) distribution, choosing, without loss of generality, $\theta=\beta=1$. Now, this sample will be treated as if θ and β are unknown.

2. If β is unknown, estimate the shape parameter β by either the moment method estimator applied to the sample $y_1, \dots, y_n$, and given by:

$$\hat{\beta} = \frac{\pi}{\sqrt{6}\, s_Y}.$$

or by the MLE based on the original sample$X_1, \dots, X_n$. If the MLE is to be applied, then β must be ≥ 1.

3. Compute the maximum Likelihood estimator of scale parameter θ by

$$\hat{\theta} = \left[\frac{1}{n} \sum_i^n X_i^{\hat{\beta}}\right]^{\frac{1}{\hat{\beta}}}.$$

Where $\hat{\beta}$ is the estimate obtained in step 2.

4. Calculate the value of the test statistic T.

5. Steps 1 through 4 are repeated 20,000 times.

6. Sort the 20,000 test statistic values and then locate the required quantiles.

Figure 3.1 shows a flow chart of the process used to obtain the critical values for the proposed test statistic T.

Note that the test depends on the transformed sample $W_1, \dots, W_n$ where $W_i = (\frac{X_i}{\theta})^{\beta}, i = 1, \dots, n,$ and where β and θ may be replaced by their estimates when one or both are unknown.

Since $F(x; \beta, \theta) = 1 - \exp[-(\frac{x}{\theta})^{\beta}] = F(w; 1,1)$, so it is sufficient to tabulate the test quantiles for Weibull(1,1).

The quantiles of the proposed test for the three cases are given in Tables 3.1, 3.2 and 3.3 , respectively. Figures 3.1, 3.2 and 3.3 represent the graphical relationships between the quantiles of proposed test, and sample size n at difference significance level α . From these tables and figures we can see that the critical values of the proposed test monotonically decrease as sample size n increases. Also the critical values increase as the significant level α decreases.

Moreover, from the tables 3.2 and 3.3 and figures 3.3 and 3.3 we can see that the critical values of the proposed test T calculated by the MLEs method are significantly different from those calculated by a combination of both MLE and MME method.

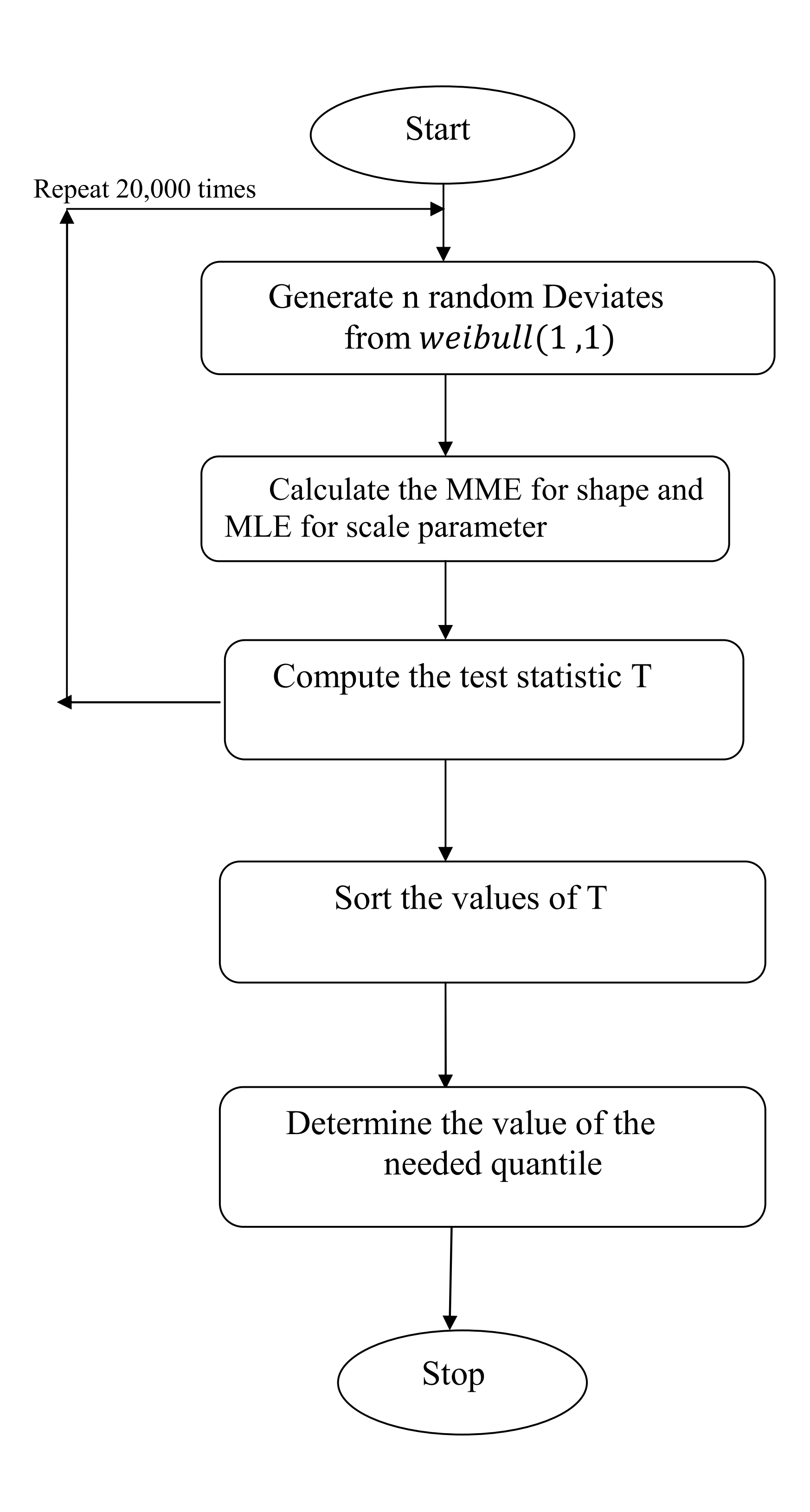


Figure 3.1 Quantile Generation

Table 3.1 Simulated quantiles of the proposed test when the shape parameter β =2

N	1-α						
	0.5	0.75	0.90	0.95	0.975	0.990	0.995
5	0.1898	0.4137	0.6377	0.7747	0.8953	1.4888	2.2840
10	0.0818	0.2081	0.3753	0.5079	0.6609	1.1315	1.5368
15	0.0555	0.1434	0.2696	0.3637	0.5120	0.7975	1.0552
20	0.0407	0.1116	0.2114	0.2928	0.4054	0.6140	0.8365
25	0.0326	0.0888	0.1759	0.2526	0.3329	0.4794	0.6114
30	0.0269	0.0750	0.1496	0.2108	0.2813	0.4294	0.5353
35	0.0231	0.0661	0.1281	0.1835	0.2406	0.3478	0.4569
40	0.0201	0.0575	0.1140	0.1630	0.2251	0.3112	0.3719
45	0.0177	0.0510	0.1055	0.1489	0.1990	0.2741	0.3645
50	0.0160	0.0469	0.0964	0.1344	0.1783	0.2534	0.3221
55	0.0151	0.0442	0.0881	0.1244	0.1680	0.2292	0.2794
60	0.0140	0.0395	0.0790	0.1131	0.1514	0.2132	0.2621
65	0.0129	0.0368	0.0749	0.1050	0.1404	0.193	0.2446
70	0.0119	0.0338	0.0703	0.1004	0.1309	0.1773	0.2207
75	0.0113	0.0326	0.0650	0.0906	0.1265	0.1633	0.2095
80	0.0103	0.0303	0.0604	0.0847	0.1168	0.1652	0.1927
85	0.0096	0.0290	0.0582	0.0825	0.1099	0.1481	0.1844
90	0.0092	0.0278	0.0557	0.0787	0.1052	0.1423	0.1662
95	0.0090	0.0259	0.0525	0.0745	0.1017	0.1295	0.1708
100	0.0084	0.0248	0.0493	0.0713	0.0945	0.1236	0.1521

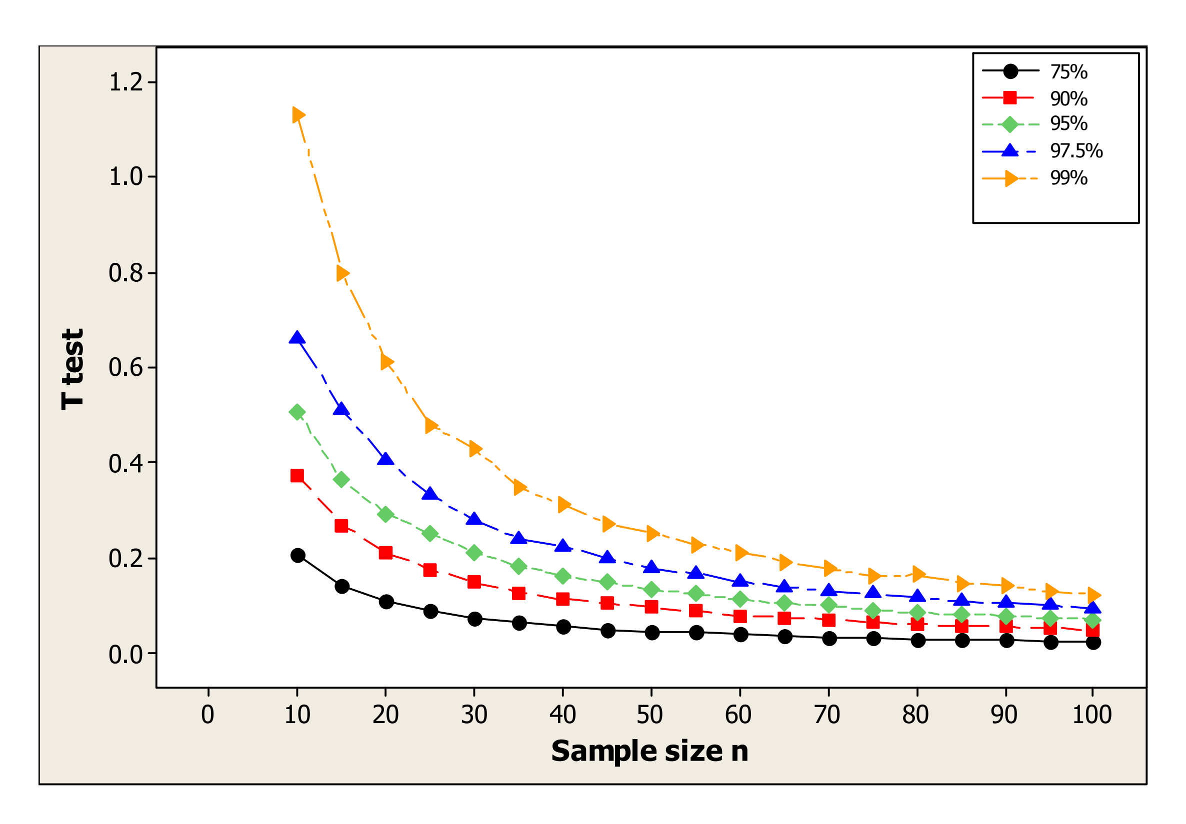


Figure 3.2 Critical values of the new test, when β =2 for different sample sizes.

Table 3.2 Simulated quantiles of the proposed test when both parameters are unknown and estimated by a mixture of MME and MLE.

N	$1-\alpha$						
	0.5	0.75	0.90	0.95	٠.975	0.990	0.995
5	0.0677	0.1826	0.3087	0.4698	0.7803	1.1746	1.3798
10	0.0570	0.1555	0.3521	0.6754	1.0893	1.9843	3.1488
15	0.0465	0.1388	0.3170	0.5456	0.9579	1.7571	2.3802
20	0.0404	0.1225	0.2660	0.4617	0.7273	1.3083	2.0017
25	0.0346	0.1064	0.2374	0.3873	0.6244	0.9731	1.3592
30	0.0313	0.0976	0.2076	0.3393	0.5045	0.8441	1.0810
35	0.0281	0.0841	0.1889	0.2886	0.4208	0.7212	0.9626
40	0.0261	0.0783	0.1683	0.2673	0.3869	0.5829	0.8352
45	0.0237	0.0726	0.1560	0.2377	0.3434	0.5211	0.6753
50	0.0224	0.0646	0.1421	0.2172	0.3088	0.4844	0.5852
55	0.0207	0.0593	0.1342	0.2042	0.2914	0.4058	0.5481
60	0.0190	0.0561	0.1228	0.1901	0.2633	0.3661	0.4845
65	0.0182	0.0539	0.1192	0.1762	0.2496	0.3699	0.4497
70	0.0168	0.0503	0.1086	0.1652	0.2295	0.3333	0.4242
75	0.0160	0.0469	0.1051	0.1497	0.2072	0.3054	0.3843
80	0.0150	0.0455	0.0958	0.1489	0.1960	0.2781	0.3557
85	0.0144	0.0418	0.0909	0.1339	0.1937	0.2590	0.3398
90	0.0136	0.0409	0.0869	0.1307	0.1772	0.2460	0.3164
95	0.0128	0.0391	0.0840	0.1241	0.1732	0.2371	0.2940
100	0.0125	0.0369	0.0804	0.1164	0.1613	0.2152	0.2721
200	0.0068	0.0201	0.0430	0.0613	0.0819	0.1109	0.1372
500	0.0029	0.0087	0.0172	0.0250	0.0329	0.0457	0.0552

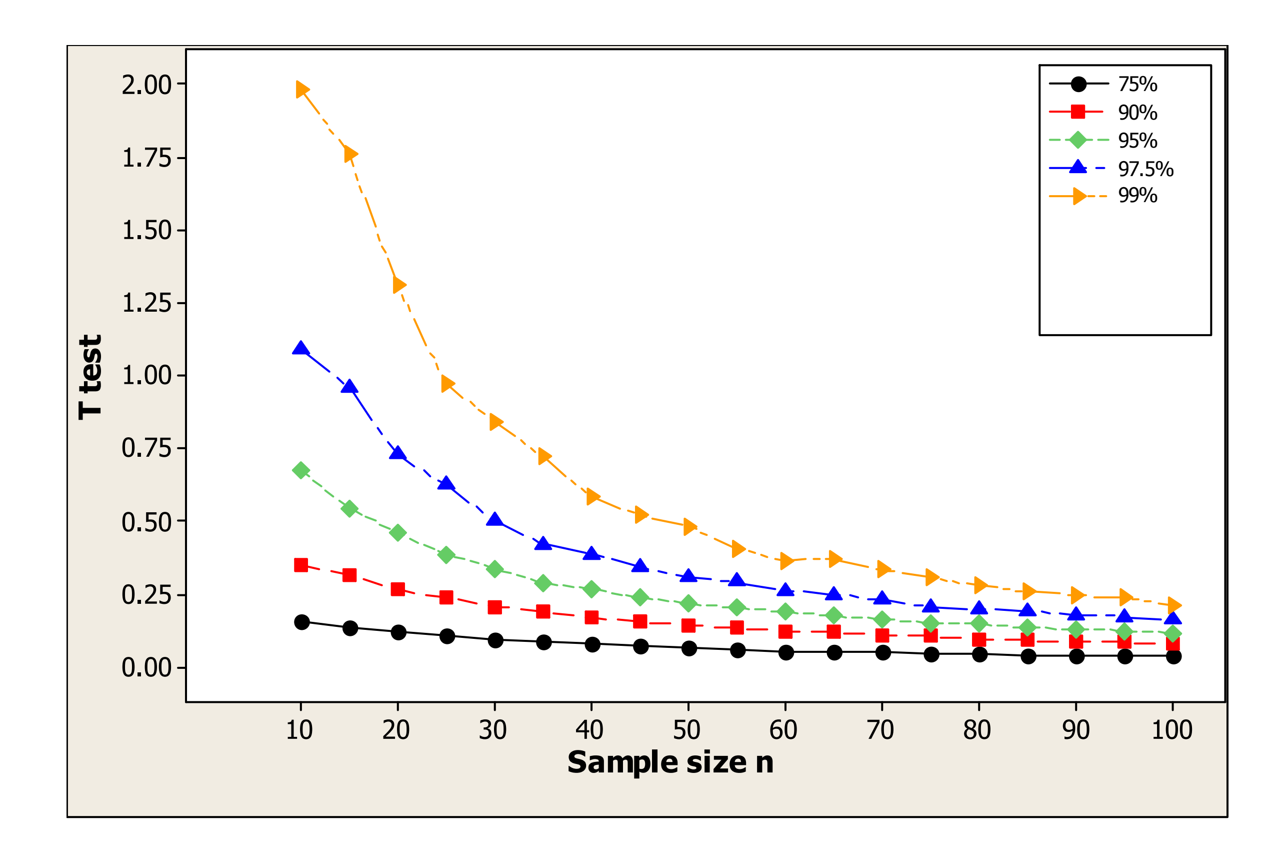


Figure 3.3 Critical Values of the new test, parameters unknown(mix), for different sample sizes, the Weibull distribution.

Table 3.3 Simulated quantiles of the proposed test when both parameters are unknown and estimated by MLE.

N	1-α						
	0.5	0.75	0.90	0.95	0.975	0.990	0.995
5	0.0113	0.0331	0.0716	0.0988	0.1190	0.1359	0.1436
10	0.0084	0.0253	0.0525	0.0801	0.1091	0.1429	0.1786
15	0.0078	0.0219	0.0457	0.0684	0.0947	0.1332	0.1558
20	0.0070	0.0201	0.042	0.0614	0.0834	0.1147	0.1498
25	0.0062	0.0182	0.0365	0.0550	0.0761	0.1112	0.1475
30	0.0055	0.0166	0.0337	0.0492	0.0665	0.0940	0.1167
35	0.0052	0.0149	0.0300	0.0450	0.0622	0.0888	0.1120
40	0.0049	0.0136	0.0281	0.0410	0.0552	0.0809	0.1033
45	0.0044	0.0125	0.0257	0.0384	0.0525	0.0783	0.0973
50	0.0041	0.0116	0.0246	0.0360	0.0494	0.0711	0.0877
55	0.0039	0.0111	0.0227	0.0334	0.0444	0.0651	0.0877
60	0.0037	0.0104	0.0216	0.0309	0.0419	0.0613	0.0831
65	0.0034	0.0099	0.0202	0.0290	0.0395	0.0579	0.0686
70	0.0032	0.0092	0.0193	0.0279	0.0381	0.0538	0.0714
75	0.0031	0.0087	0.0182	0.0260	0.0357	0.0513	0.0641
80	0.0029	0.0083	0.0171	0.0254	0.0338	0.0500	0.0600
85	0.0027	0.0079	0.0166	0.0234	0.0325	0.0487	0.0581
90	0.0027	0.0078	0.0155	0.0227	0.0315	0.0439	0.0564
95	0.0025	0.0075	0.0153	0.0219	0.0298	0.0435	0.0586
100	0.0024	0.0071	0.0149	0.0214	0.0287	0.0405	0.0533

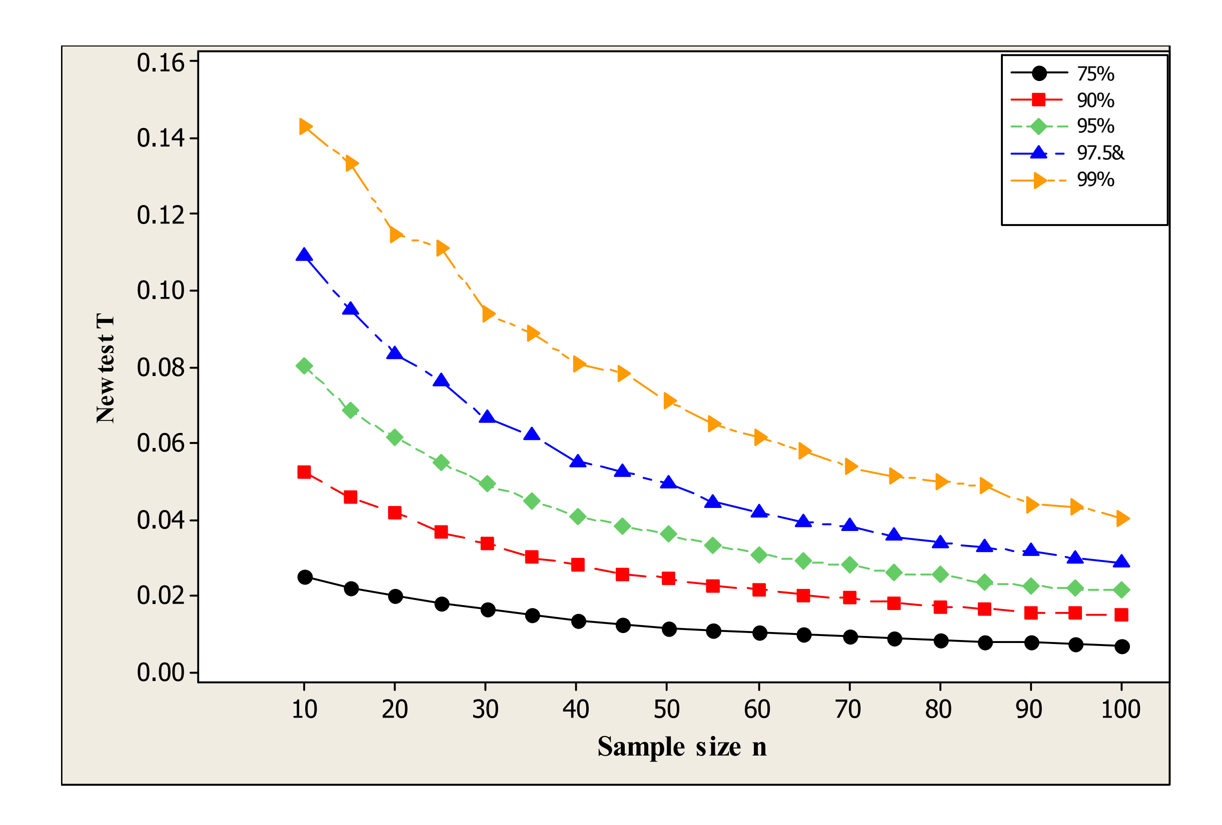


Figure 3. 4 Critical values of new test, parameters are unknown(mle), for different sample of sizes, the Weibull distribution.

3.4 Power Study

The power of a goodness of fit test is defined as the probability of rejecting the null hypothesis H_0, when H_0 in fact it is false. This is denoted by $1-\beta$, where β is the probability of type II error. The power of the test is the key to measure the performance of the test. The higher the power of a test is, the lower the probability of accepting a distribution when it is false.

The power computation procedure is similar to the procedure used to calculate the quantiles of the proposed test with first step being generating a random sample from a designated alternative distribution. That is the samples are generated from a distribution other than the hypothesized distribution, which is the Weibull in our case. To compute the simulated power, at $\alpha = 0.05$ when testing, for example , against gamma(2,1) distribution, a random sample of size n, say n=20, is simulated from gamma (2 ,1) and the value of the test statistic T is computed. This value is compared to the test quantile $T_{0.05}$= 0.4559, see Table 3.2. If the computed value of T is greater than 0.4559, the test is rejected. This procedure is repeated 10,000 times and the power of the test is the proportion of rejections.

The power of the proposed test statistic will be computed at significance levels α = 0.01, 0 .05 and 0.1 for samples of sizes n = 10, 15, 20, 25, 30, 40, 50 and 100.

Table 3.4 and table 3.5 give the cut points used in this power study when the shape parameter β is known and when it is unknown and estimated by MLE method, respectively. Table 3.6 give the cut points when both parameters are unknown and estimated by a mixture of MME and MLE.

Table 3.4 Cut points for test statistic at significance level α =0.01, 0.05, and 0.1 when β =2.

N	Significance levels α		
	0.01	0.05	0.1
10	1.1315	0.5079	0.3753
15	0.7975	0.3637	0.2696
20	0.6140	0.2928	0.2114
25	0.4794	0.2526	0.1759
30	0.4294	0.2108	0.1496
40	0.3112	0.1630	0.1140
50	0.2543	0.1344	0.0964
100	0.1236	0.0713	0.0493

Table 3.5 Cut points for test statistic at significance level α =0.01, 0.05, and 0.1 when both parameters are unknown and estimated by (MLE).

N	Significance levels α		
	0.01	0.05	0.1
10	0.1429	0.0801	0.0525
15	0.1332	0.0684	0.0457
20	0.1147	0.0614	0.042
25	0.1112	0.0550	0.0365
30	0.0940	0.0492	0.0337
40	0.0809	0.0410	0.0281
50	0.0711	0.0360	0.0246
100	0.0450	0.0214	0.0149

Table 3.6 Cut points for test statistic at significance level α =0.01, 0.05, and 0.1 when both parameters are unknown and estimated by (MIX).

N	Significance levels α		
	0.01	0.05	0.1
10	1.9843	0.6754	0.3521
15	1.7571	0.5456	0.3170
20	1.3083	0.4617	0.2660
25	0.9731	0.3873	0.2374
30	0.8441	0.3393	0.2076
40	0.5829	0.2673	0.1683
50	0.4844	0.2172	0.1421
100	0.2152	0.1164	0.0804

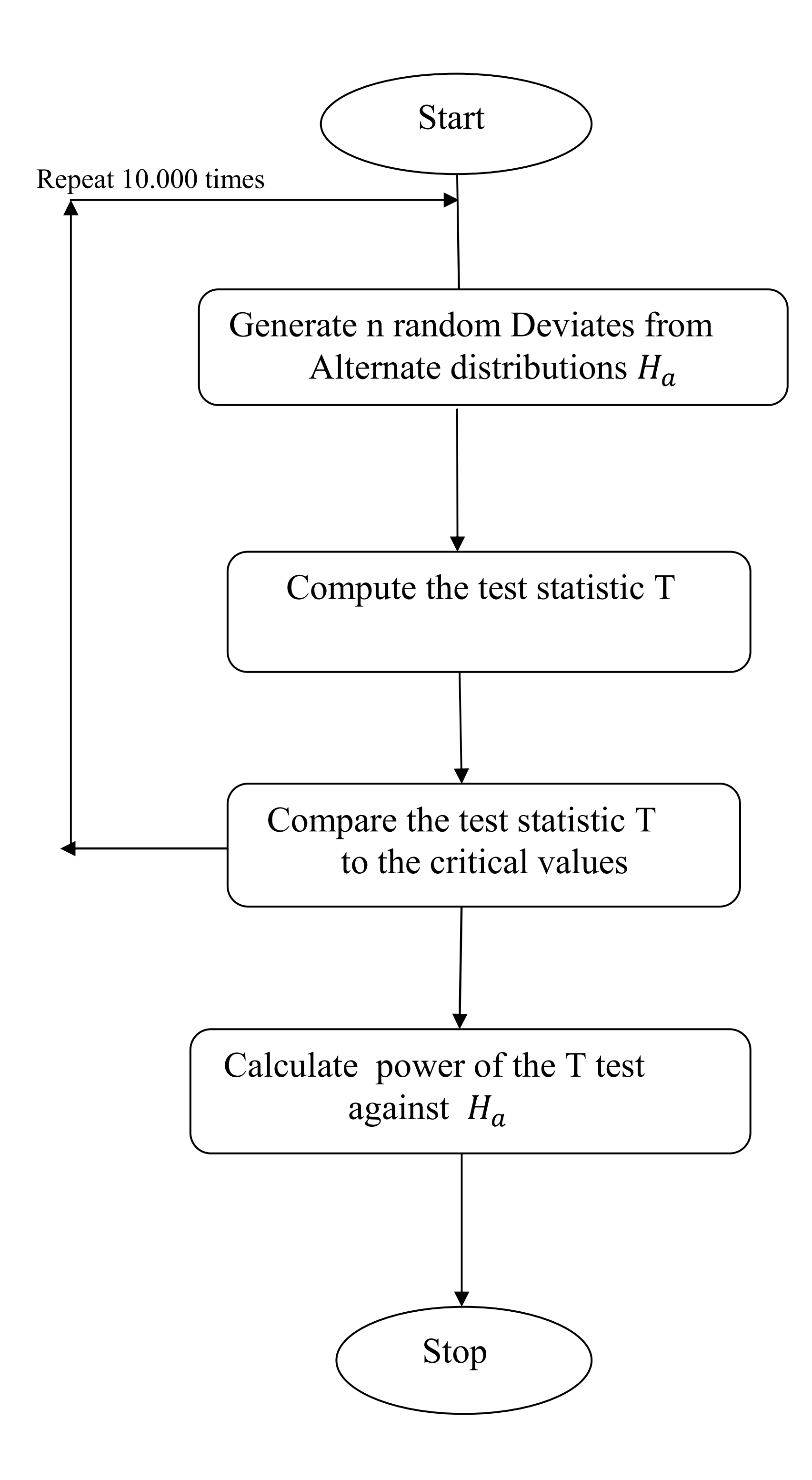


Figure 3.5 Flow Chart for the Power computation process

The power of the proposed test is computed when testing against numerous alternatives. These alternatives include gamma, normal, Cauchy, lognormal, beta , chi-square, uniform, Weibull, and Pareto distributions. We test against the Weibull distribution to verify the validity of the test. The reason behind this choice of alternatives is that these families of distributions cover a wide range of distributions, and they are commonly used in power studies concerning goodness of fit tests for the Weibull distribution.

The following Specific distributions from the above families of distributions are selected here as alternative distributions in our power study:

1. Gamma distribution (2 , 1).
2. Gamma distribution (4 , 1).
3. Weibull distribution (2 , 1).
4. LoNormal distribution(0 , 1).
5. Folded Normal distribution (10 , 1).
6. Chi-Square distribution with 1 degree of freedom.
7. Chi-Square distribution with 4 degrees of freedom.
8. Folded Cauchy distribution (0 , 1).
9. Floded Student distribution with 1 degree of freedom.
10. Beta distribution(1,1).
11. Uniform distribution (0 ,1).
12. Uniform distribution (10 , 15).
13. Pareto distribution (2 , 1).

The power of the proposed test will be computed for the following cases:

1) The shape parameter β is known, while the scale parameter θ is unknown and estimated by the MLE.
2) The shape and the scale are both unknown and are estimated by the MLE.
3) The shape and the scale are both unknown and are estimated by a combination of MME and MLE, which denoted by MIX method.

For all cases the powers will be calculated for samples of size n = 10(5)30,40,50 and 100 and at significance levels $\alpha = 0.01$, 0.05 and 0.1, except for the first case, the power is calculated for $\alpha = 0.05$ only.

Looking at any table of power study , it is clear that the powers increases as the significance level of the test increasing, also the powers increases as the sample size gets larger.

Power when β is known

For the case, when the shape parameter β is known and equal 2, for example, power results for the proposed test T are summarized in Table 3.7. We can see that the powers result is extremely high for the uniform(10.15) and normal(10,1) alternatives. In fact, the power is 1 even with small sample size 10 and significance level 0.05. A different story was appeared for the gamma(4,1) and uniform(0,1) alternatives. The powers result are extremely poor for these alternatives, there is no power greater than 0.09 even with large sample size n=100.

The proposed test statistic T has excellent power when testing against Weibull(1,1), lognormal, Cauchy, $\chi^2(1)$ and Pareto alternatives, the powers are more than 0.88 for

n = 20. In general , it can be noted that when the shape parameter is known, the power is high in most of cases.

Power when β and θ are unknown and estimated by MLE

For the case, when the shape and scale parameters are unknown and estimated by MLE. power results for the proposed test T are displayed in tables 3.8 and 3.9. From these tables it can be seen that there is no power greater than 0.20 even with sample size of 100 for the Gamma and χ^2 alternatives. In fact, these results are not surprising since these alternatives are similar to the Weibull. Also, it can be notde that the power is lower for the $\chi^2(4)$ than for the $\chi^2(1)$.

The best power is achieved for Cauchy, Pareto, normal and log-normal alternatives distribution .For the log-normal case, the power is more than 0.93 for sample size 100 and significance level 0.05. For the Pareto case, the power is more than 0.9 even with small sample size 15 and significance level 0.05. However, The power is reasonably high when the alternative is uniform. Especially with large sample of size (n ≥ 50).

Power when β and θ are unknown and estimated by MIX

When the shape and the scale parameters are both unknown and are estimated by a combination of MME and MLE. Tables 3.10 and 3.11 summarized the Power results for the proposed test statistics T at significance level 0.05 and 0.1, respectively. From these tables it can be noted that The proposed test statistic T has excellent power when testing against Lognormal, Cauchy, and Pareto alternatives. The power is above 0.41 when testing against log normal for sample size 20 at α = 0.05 and above 0.87 for n= 50 at α = 0.1. When testing against Cauchy, the power is equal 0.5 for sample size 20 at significance level 0.05 and above 0.85 for n = 50 at α = 0.1. The best power is

achieved when the alternative distribution is Pareto, the power was above 0.74 even with small sample, $n = 10$ and $\alpha = 0.05$.

It is interested to note that the power has considerably increased when the parameters are estimated by the MIX method compared to the power under MLE estimators. For example, the power against log-normal (0,1) for $n = 30$ and $\alpha = 0.05$ under MLE estimators is 0.37 increased to 0.59 under MIX estimators. In fact, the power has increased for all alternatives.

Table 3.7 Power Result of the Test Statistic ; α = 0.05 ; Shape parameter is known (β=2) ; H_0 : $Weibull\ distributio$ 2s. H : αot=$er\ distributio$

Alternative distribution							
N	W(2,1)	W(1,1)	LN(0,1)	N(10,1)	GM(2,1)	GM(4,1)	CU(0,1)
10	0.051	0.654	0.527	1.000	0.264	0.066	0.90
15	0.050	0.801	0.875	1.000	0.373	0.07	0.969
20	0.052	0.884	0.943	1.000	0.46	0.072	0.992
25	0.047	0.925	0.974	1.000	0.528	0.070	0.997
30	0.048	0.952	0.990	1.000	0.608	0.076	0.998
50	0.051	0.99	0.999	1.000	0.780	0.079	0.999
100	0.050	0.999	1.000	1.000	0.959	0.078	1.000
N	$\chi_{(1)}$	$\chi_{(H)}$	ST(1)	PR(2,1)	BE(1,1)	U(0,1)	U(10,15)
10	0.868	0.270	0.900	0.797	0.081	0.081	1.000
15	0.944	0.371	0.972	0.927	0.086	0.082	1.000
20	0.97	0.468	1.000	0.962	0.081	0.080	1.000
25	0.983	0.525	0.997	0.986	0.071	0.074	1.000
30	0.991	0.600	0.999	0.995	0.085	0.077	1.000
50	0.998	0.783	1.000	1.000	0.080	0.084	1.000
100	1.000	0.958	1.000	1.000	0.089	0.082	1.000

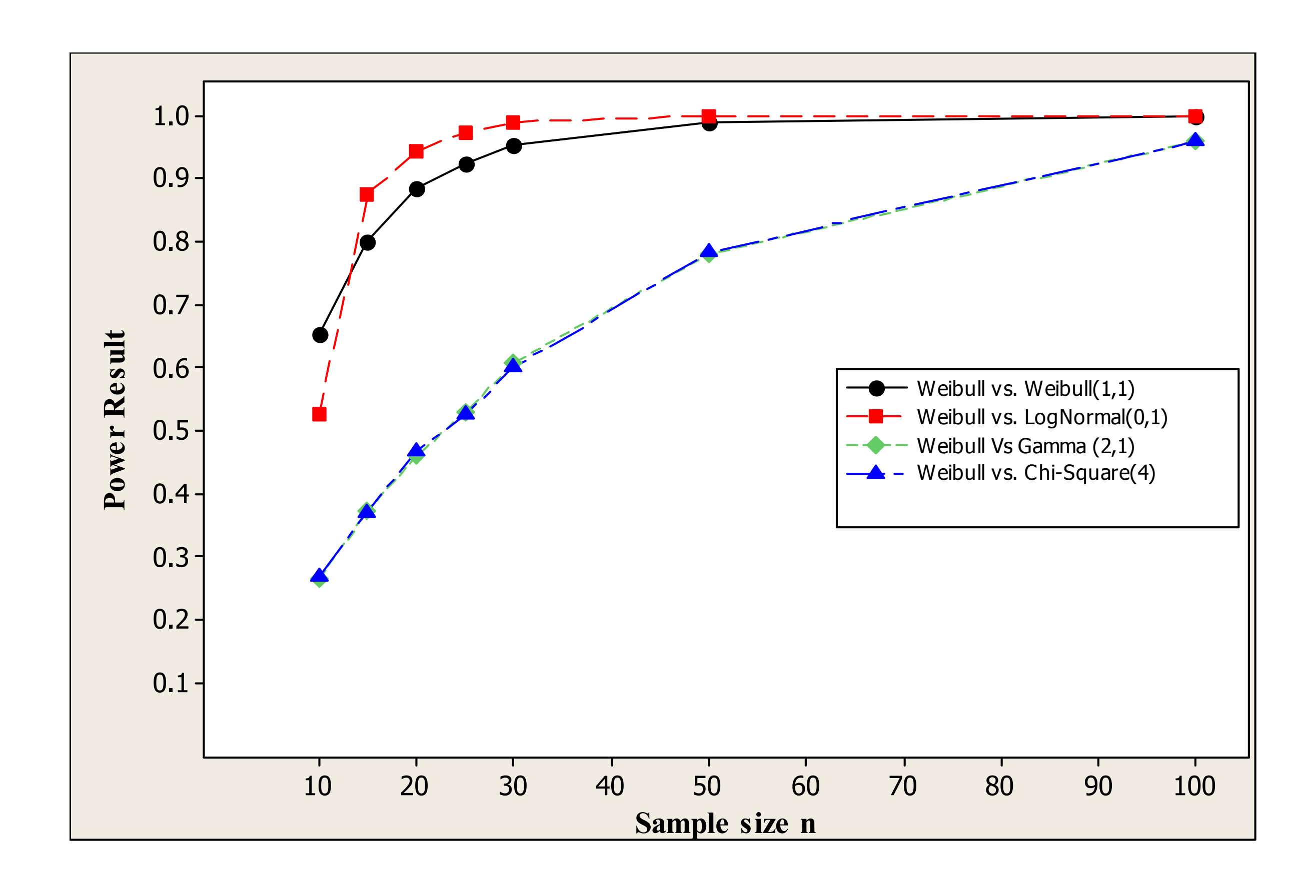


Figure 3.6 Power Results of The new test T ; shape parameter is known; α= 0.05

Table 3.8 Power Result; α = 0.05 ; both parameter are unknown (MLE) ;
H_0 : *Weibull distributio*□ 2*s*. *H* : α□*ot=er distributio*□

N	Alternative distribution					
	W(2,1)	LN(0,1)	N(10,1)	GM(2,1)	GM(4,1)	CU(0,1)
10	0.049	0.070	0.049	0.040	0.040	0.132
15	0.050	0.151	0.084	0.046	0.059	0.235
20	0.046	0.219	0.114	0.046	0.068	0.31
25	0.048	0.288	0.149	0.054	0.080	0.379
30	0.0509	0.367	0.182	0.057	0.1009	0.448
40	0.053	0.506	0.259	0.063	0.127	0.533
50	0.0506	0.62	0.31	0.072	0.16	0.599
100	0.0491	0.936	0.624	0.1185	0.132	0.823

N	$\chi_{(1)}$	$\chi_{(H)}$	ST(1)	PR(2,1)	BE(1,1)	U(0,1)	U(10,15)
10	0.085	0.040	0.132	0.543	0.15	0.148	0.018
15	0.088	0.041	0.241	0.852	0.182	0.185	0.042
20	0.092	0.056	0.30	0.961	0.20	0.208	0.062
25	0.095	0.049	0.372	0.989	0.23	0.239	0.081
30	0.098	0.056	0.441	0.998	0.259	0.265	0.123
40	0.109	0.064	0.527	0.999	0.319	0.322	0.217
50	0.126	0.068	0.61	1.0	0.358	0.361	0.31
100	0.175	0.115	0.829	1.0	0.602	0.608	0.796

Table 3.9 Power of the Test Statistic ; α = 0.1 ; both parameter are unknown (MLE) H_0 : *Weibull distributio*� 2*s*. H : α�*ot=er distributio*�

N	Alternative distribution					
	W(2,1)	LN(0,1)	N(10,1)	GM(2,1)	GM(4,1)	CU(0,1)
10	0.104	0.108	0.115	0.10	0.101	0.227
15	0.107	0.274	0.176	0.107	0.124	0.351
20	0.10	0.355	0.219	0.11	0.149	0.41
25	0.105	0.476	0.28	0.122	0.185	0.511
30	0.107	0.547	0.318	0.126	0.21	0.545
40	0.102	0.682	0.405	0.141	0.252	0.645
50	0.095	0.78	0.468	0.152	0.29	0.697
100	0.097	0.974	0.772	0.221	0.490	0.884

N	$\chi_{(1)}$	$\chi_{(H)}$	ST(1)	PR(2,1)	BE(1,1)	U(0,1)	U(10,15)
10	0.148	0.096	0.227	0.713	0.247	0.238	0.066
15	0.156	0.112	0.35	0.933	0.27	0.272	0.116
20	0.15	0.11	0.43	0.987	0.30	0.294	0.17
25	0.158	0.122	0.508	0.998	0.353	0.351	0.237
30	0.161	0.127	0.549	0.999	0.366	0.362	0.296
40	0.179	0.149	0.644	1.0	0.426	0.435	0.437
50	0.178	0.153	0.704	1.0	0.474	0.464	0.56
100	0.252	0.227	0.880	1.0	0.712	0.715	0.935

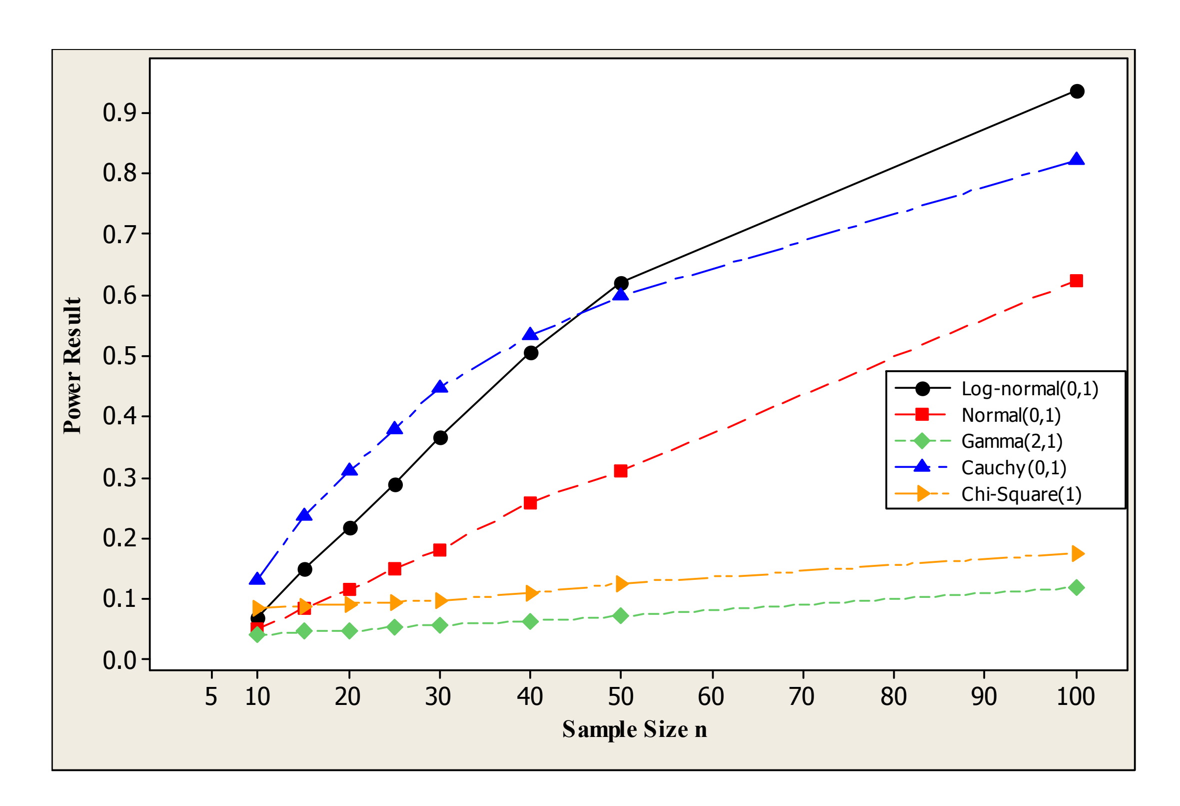


Figure 3.7 Power Results of The new test T with unknown parameter estimated via (MLE) ; α= 0.05

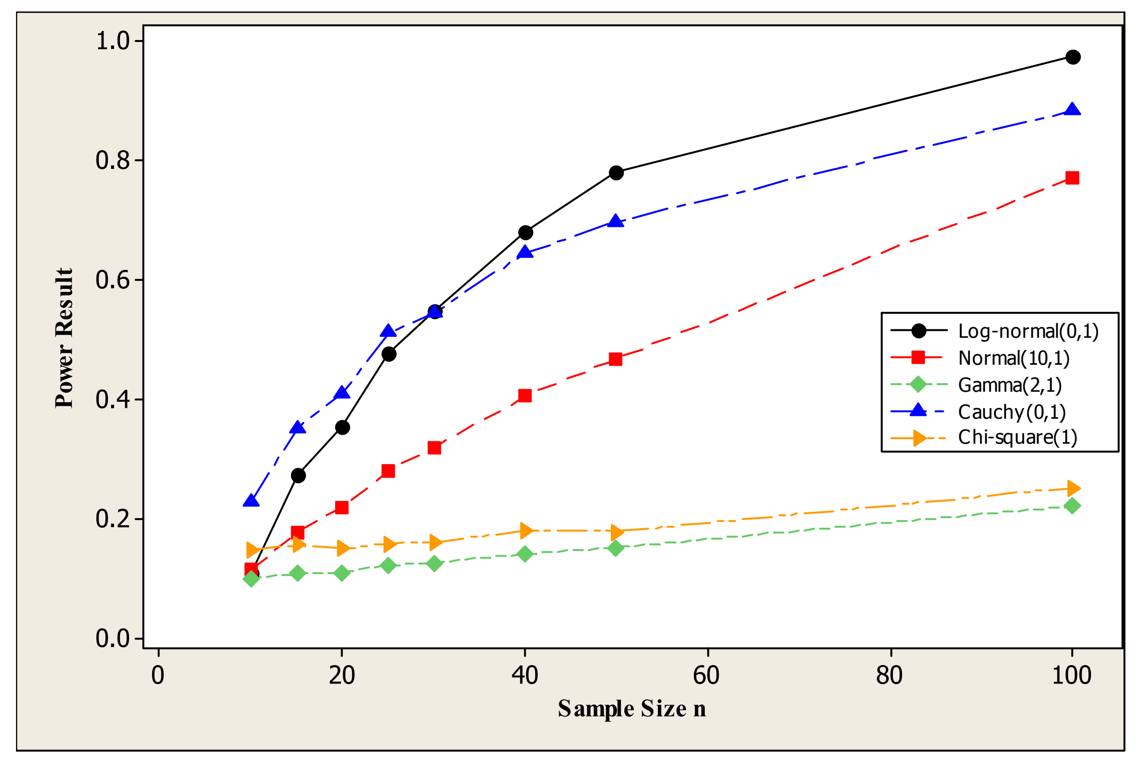


Figure 3.8 Power Results of The new test T with unknown parameter estimated via (MLE) ; α= 0.1

Table 3.10 Power of the Test Statistic ; α = 0.05 ; both parameter are unknown (MIX) H_0 : $Weibull\ distributio□\ 2s.\ H$: $α□ot{=}er\ distributio□$

Alternative distribution

N	W(2,1)	LN(0,1)	N(10,1)	GM(2,1)	GM(4,1)	CU(0,1)
10	0.050	0.212	0.143	0.079	0.106	0.274
15	0.048	0.332	0.211	0.096	0.145	0.407
20	0.049	0.415	0.251	0.107	0.170	0.50
25	0.050	0.502	0.322	0.120	0.206	0.574
30	0.049	0.589	0.362	0.132	0.228	0.643
40	0.049	0.709	0.460	0.145	0.280	0.731
50	0.050	0.809	0.537	0.162	0.341	0.807
100	0.052	0.978	0.815	0.246	0.550	0.960

N	Chi(1)	Chi(4)	ST(1)	PR(2,1)	BE(1,1)	U(0,1)	U(10,15)
10	0.025	0.077	0.290	0.74	0.033	0.035	0.079
15	0.020	0.099	0.407	0.922	0.002	0.002	0.098
20	0.017	0.106	0.499	0.979	0.012	0.015	0.117
25	0.050	0.117	0.570	0.996	0.029	0.026	0.140
30	0.02	0.126	0.65	0.999	0.047	0.045	0.160
40	0.028	0.145	0.732	1.000	0.097	0.095	0.232
50	0.044	0.168	0.805	1.000	0.167	0.167	0.298
100	0.102	0.243	0.955	1.000	0.519	0.518	0.638

Table 3.11 Power of the Test Statistic ; α = 0.1 ; both parameter are unknown (MIX) $H_0 : Weibull\ distribution\ vs.\ H_1 : another\ distribution$

N	Alternative distribution					
	W(2,1)	LN(0,1)	N(10,1)	GM(2,1)	GM(4,1)	CU(0,1)
10	0.104	0.316	0.238	0.141	0.187	0.375
15	0.0992	0.4375	0.3141	0.1617	0.2263	0.5013
20	0.10	0.546	0.374	0.186	0.276	0.587
25	0.1044	0.6297	0.4291	0.1919	0.306	0.6567
30	0.103	0.692	0.488	0.21	0.34	0.721
40	0.1026	0.8089	0.5784	0.2261	0.394	0.8009
50	0.098	0.878	0.650	0.258	0.446	0.853
100	0.0993	0.9981	0.8810	0.3439	0.6624	0.9697

N	$\chi_{(1)}$	$\chi_{(H)}$	ST(1)	PR(2,1)	BE(1,1)	U(0,1)	U(10,15)
10	0.070	0.143	0.376	0.84	0.056	0.058	0.152
15	0.0631	0.1644	0.4948	0.9601	0.0712	0.070	0.1896
20	0.069	0.179	0.588	0.993	0.113	0.116	0.228
25	0.0718	0.1933	0.6641	0.9987	0.1415	0.150	0.2543
30	0.076	0.214	0.710	0.999	0.19	0.19	0.29
40	0.1018	0.2356	0.81	1.0000	0.2816	0.2826	0.3834
50	0.11	0.256	0.85	1.0000	0.364	0.357	0.474
100	0.1898	0.3279	0.9671	1.0000	0.7063	0.7162	0.7970

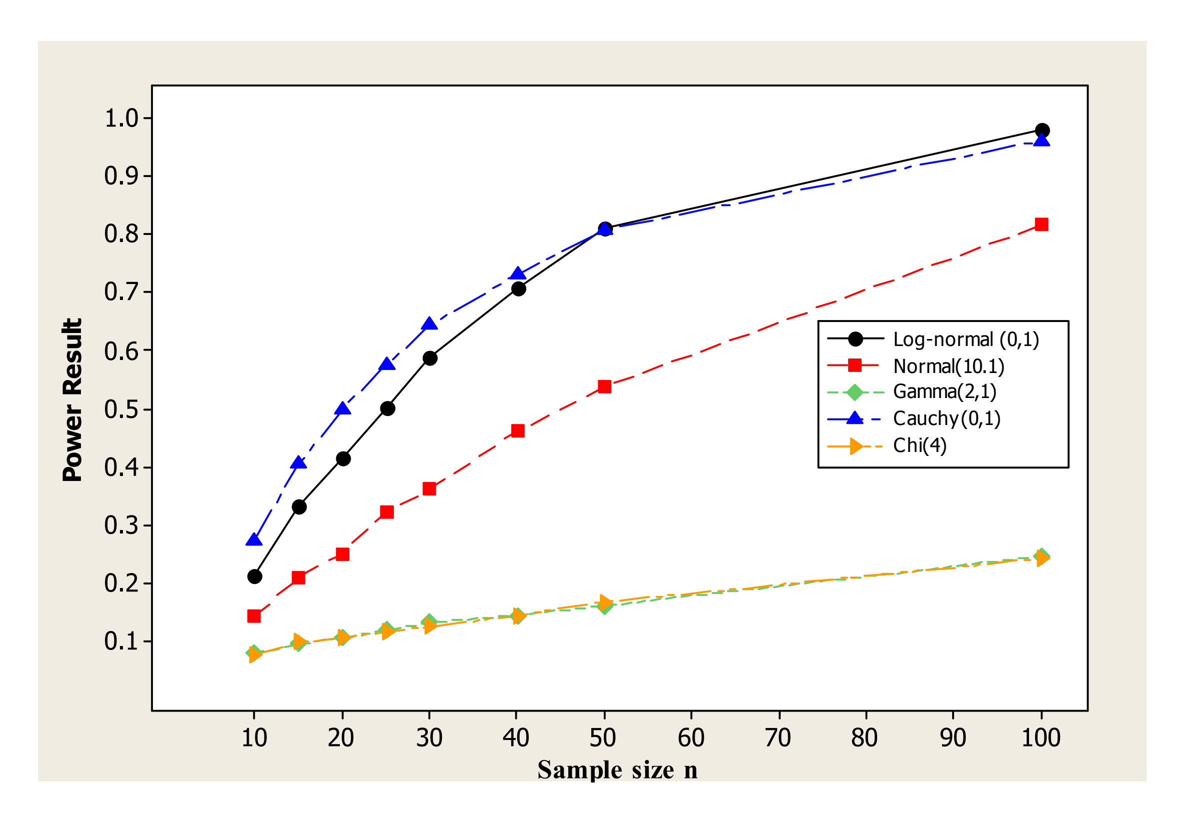


Figure 3.9 Power Results of The new test T with unknown parameter estimated via (MIX) for ; α= 0.05

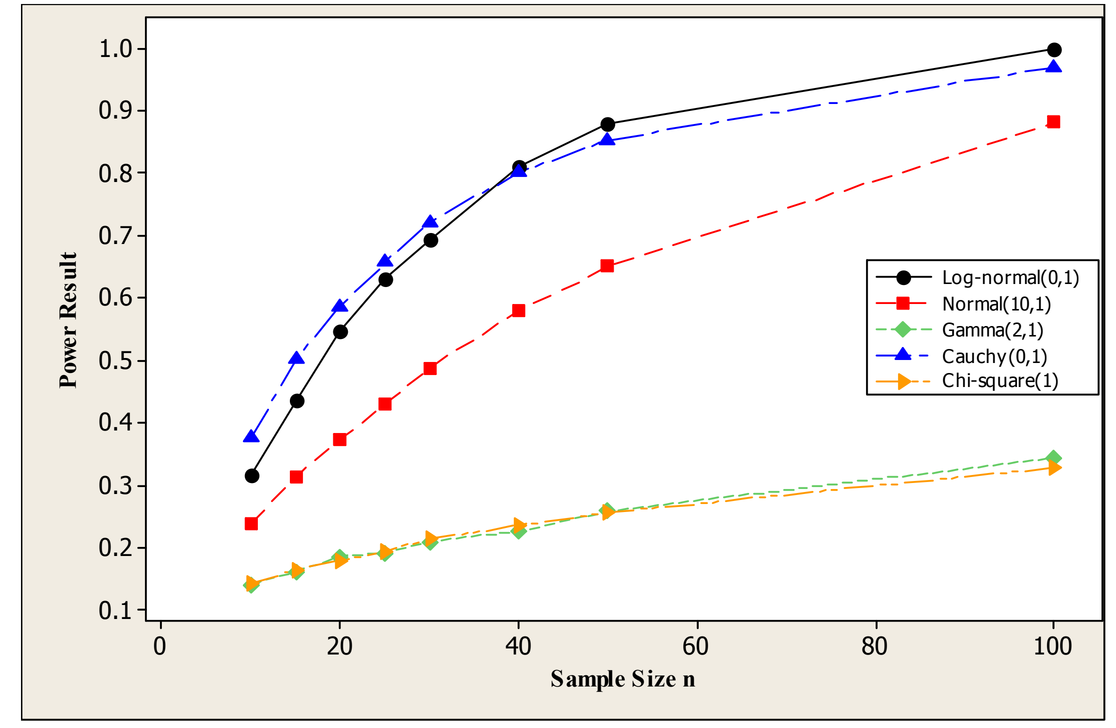


Figure 3.10 Power Results of The new test T with unknown parameter estimated via (MIX) for ; α= 0.1

3.5 Comparative Power Study

In this thesis, four well known goodness-of-fit tests for Weibull distribution will be compared with the proposed test. The four tests are Anderson-Darling test (A^2), Cramer von Mises test (W^2), Kolmogorov-Simrnov test (K-S) and Liao- Shimokawa test (L). Comparison is made for two cases, the first case when the shape and the scale parameters are both unknown and are estimated by the MLE and the second case when the shape and the scale parameters are both unknown and are estimated by a combination of MME and MLE methods.

In order to get the power result for the above tests, first, the critical values must be tabled for these tests for sample of size n=10(10)100 and significance level α = 0.01, 0.05and 0.1. Moreover, The critical values given by Littell et al. (1979) for the A^2, W^2 and K-S tests and critical values given by Liao and Shimokawa (1999) for L test are also listed.

Tables 3.12 and 3.13 give the critical values of above tests for samples of size n= 10(10)100 , at significance level α = 0.01, 0,05 and 0.1. However, from these tables it can be see that the critical values of A^2, W^2 , K-S and L calculated are approximately equal to those given by Littell et.al. and Liao and Shimokawa. Tables 3.14 and 3.15 show the powers of the competing tests when the MLEs are used to estimate the parameters for samples of size n= 20,30 and 50 at the significance level α =0.05 and 0.1. Moreover, the average power for all sample sizes for all alternatives are calculated.

Tables 3.16 and 3.17 show the powers of the competing tests when a combination of MME and MLE denoted by (MIX) are used to estimate the parameters for the sample of size n= 20,30 and 50 at the significance level α =0.05 and 0.1. Moreover, the average power for all sample sizes for all alternatives are calculated.

From the results in tables 3.14 , 3.15, 3.16 and 3.17 and the average power table 3.18 the power comparisons concluded the following:

1) The proposed test with MIX estimators is the most powerful test over all other tests for almost all alternatives under consideration.
2) For all tests considered T, A^2, W^2, K-S and L the power under MIX method of estimation is superior to that under MLE method.
3) The proposed test T with MLE case has almost equivalent power to the Anderson-Darling (A^2) test when testing against normal, log-normal and Pareto alternatives, but for MIX case the proposed test is more powerful than Anderson-Darling (A^2) for all alternatives except for Pareto and $\chi^2_{(1)}$.
4) In comparison to the Liao- Shimokawa test (L), the proposed test T with MLE case, proves higher power when testing against log-normal, normal, Pareto and $\chi^2_{(H)}$ alternatives . Whereas L has better power when testing against Cauchy and $\chi^2_{(1)}$ alternatives.
5) The proposed test T for MLE case is superior to Kolmogorov- Smirnov test for all alternatives and to Cramer-von Mises test for almost all alternatives.

Table 3.12. Quantiles of the tests for MIX method.

Test	N	α		
		0.1	0.05	0.01
k-s	10	0.2938	0.3208	0.3892
	20	0.2163	0.2420	0.2911
	30	0.1821	0.2023	0.2444
	40	0.1585	0.1752	0.2167
	50	0.1430	0.1591	0.1903
	60	0.1308	0.1450	0.1753
	70	0.1231	0.1363	0.1640
	80	0.1146	0.1283	0.1541
	90	0.1083	0.1188	0.1444
	100	0.1022	0.1136	0.1388
		0.1576	0.2064	0.3425
W^2	10	0.1796	0.2293	0.3906
	20	0.1840	0.2445	0.4043
	30	0.1846	0.2509	0.3887
	40	0.1921	0.2507	0.4075
	50	0.1912	0.2560	0.4028
	60	0.1943	0.2557	0.4116
	70	0.1943	0.2508	0.4246
	80	0.1978	0.2609	0.3950
	90	0.2014	0.2539	0.4106
	100			
A^2	10	0.8857	1.1193	1.815
	20	0.9517	1.2412	1.9448
	30	1.0151	1.3134	2.0445
	40	1.0616	1.3552	2.0436
	50	1.0530	1.3678	2.1567
	60	1.0639	1.3260	2.0761
	70	1.0724	1.3517	2.2166
	80	1.0777	1.4104	2.1272
	90	1.0878	1.3999	2.1281
	100	1.0778	1.3914	2.2601
L	10	1.2987	1.4650	2.0020
	20	1. 1964	1.3835	1.9041
	30	1.1633	1.3308	1.7774
	40	1.1352	1.2841	1.7053
	50	1.1130	1.2712	1.6015
	60	1.1030	1.2302	1.603
	70	1.0712	1.23	1.5419
	80	1.0682	1.2148	1.4994
	90	1.0689	1.2190	1.5139
	100	1.0615	1.2064	1.5149

Table 3.13. Quantiles of the tests for MLE method.

Test	N	α		
		0.1	0.05	0.01
k-S	10	0.2403(.240)	0.2605(.260)	0.2995(.300)
	20	0.1751(.175)	0.1889(.191)	0.2190(.220)
	30	0.1444(.144)	0.1568(.156)	0.1808(.179)
	40	0.1259(.125)	0.1369(.136)	0.1580(.158)
	50	0.1131	0.1228	0.1433
	60	0.1037	0.1127	0.1302
	70	0.0963	0.1039	0.1203
	80	0.0900	0.0974	0.1139
	90	0.0847	0.0921	0.1067
	100	0.0800	0.0869	0.1002
W^2	10	0.0995(.101)	0.1205(.120)	0.1662(.164)
	20	0.1005(.101)	0.1219(.123)	0.1683(.172)
	30	0.1009(.101)	0.1243(.122)	0.1716(.172)
	40	0.1007(.101)	0.1232(.123)	0.1730(173)
	50	0.1024	0.1239	0.1747
	60	0.1002	0.1224	0.1757
	70	0.1015	0.1471	0.1720
	80	01005	0.1234	0.1770
	90	0.1024	0.1238	0.1744
	100	0.0999	0.1235	0.1709
A^2	10	0.6186(.616)	0.7196(.730)	0.9975(988)
	20	0.6162(.625)	0.7375(.744)	1.0407(1.012)
	30	0.6233(.626)	0.7509(.739)	1.0009(1.003)
	40	0.6357(.627)	0.7420(.739)	1.0643(1.001)
	50	0.6246	0.7504	1.0094
	60	0.6353	0.7505	1.0301
	70	0.6227	0.7498	1.0222
	80	0.6309	0.7433	1.0300
	90	0.6325	0.7668	1.0257
	100	0.6319	0.7522	1.0411
L	10	1.137(1.145)	1.237(1.241)	1.501(1.509)
	20	1.0142(1.015)	1.103(1.100)	1.310(1.313)
	30	0.9422(.954)	1.033(1.033)	1.2772(1.220)
	40	0.9236(.916)	0.989(.992)	1.1584(1.166)
	50	0.8877(.890)	0.961(.965)	1.1122(1.131)
	60	0.8616	0.951	1.1078
	70	0.8599	0.914	1.0833
	80	0.8225	0.933	1.0431
	90	0.8521	0.893	1.0306
	100	0.8165(.824)	0.906(.895)	1.0527(1.045)

Table 3.14 power comparisons of T test ; α = .05 ; MLE

N	test	Alternatives								
		Weibll (2,1)	Gamma (2,1)	Normal (10,1)	Lo-nom (0,1)	Cuchy (0,1)	Parto (2,1)	$\chi_{(1)}$	$\chi_{(H)}$	Average power
20	T	0.046	0.046	0.114	0.219	0.31	0.96	0.092	0.056	.230
	A^2	0.069	0.06	0.125	0.216	0.37	0.95	0.086	0.068	.243
	W^2	0.056	0.067	0.121	0.208	0.34	0.94	0.082	0.061	.234
	K-S	0.068	0.061	0.107	0.174	0.30	0.88	0.078	0.074	.217
	L	0.046	0.046	0.09	0.174	0.35	0.90	0.109	0.055	.21
30	T	0.052	0.052	0.19	0.37	0.44	0.998	0.106	0.057	.283
	A^2	0.053	0.055	0.18	0.35	0.54	0.996	0.103	0.064	.292
	W^2	0.047	0.06	0.18	0.32	0.49	0.992	0.088	0.058	.277
	K-S	0.055	0.058	0.14	0.24	0.40	0.983	0.08	0.06	.252
	L	0.053	0.054	0.17	0.30	0.52	0.988	0.13	0.06	.284
50	T	0.050	0.072	0.31	0.62	0.60	1.00	0.126	0.068	.355
	A^2	0.051	0.09	0.30	0.56	0.76	1.00	0.119	0.085	.370
	W^2	0.049	0.084	0.26	0.49	0.73	1.00	0.10	0.081	.349
	K-S	0.049	0.073	0.20	0.37	0.63	0.999	0.095	0.072	.311
	L	0.053	0.075	0.27	0.51	0.758	1.00	0.14	0.075	.360

Table 3.15 power comparisons of T test ; α = .1 ; MLE

N	test	Weibll (2,1)	Gamma (2,1)	Normal (10,1)	Lo-nom (0,1)	Cuchy (0,1)	Parto (2,1)	$\chi_{(1)}$	$\chi_{(H)}$	average power
20	T	0.10	0.11	0.22	0.355	0.41	0.987	0.15	0.11	0.305
	A^2	0.13	0.117	0.208	0.326	0.47	0.977	0.15	0.122	0.312
	W^2	0.088	0.116	0.205	0.31	0.49	0.967	0.13	0.121	0.303
	K-S	0.098	0.106	0.175	0.25	0.39	0.935	0.126	0.11	0.273
	L	0.098	0.107	0.165	0.26	0.44	0.951	0.17	0.10	0.286
30	T	0.095	0.13	0.31	0.53	0.54	1.00	0.14	0.116	0.357
	A^2	0.11	0.11	0.29	0.45	0.61	1.00	0.16	0.114	0.355
	W^2	0.086	0.075	0.22	0.40	0.58	1.00	0.133	0.105	0.324
	K-S	0.107	0.108	0.24	0.35	0.50	0.99	0.13	0.102	0.315
	L	0.108	0.11	0.26	0.42	0.61	1.00	0.19	0.105	0.350
50	T	0.095	0.152	0.468	0.78	0.697	1.00	0.178	0.153	0.440
	A^2	0.106	0.159	0.417	0.68	0.816	1.00	0.20	0.162	0.442
	W^2	0.098	0.145	0.365	0.61	0.792	1.00	0.165	0.149	0.415
	K-S	0.101	0.13	0.30	0.50	0.718	1.00	0.156	0.138	0.380
	L	0.103	0.14	0.38	0.637	0.81	1.00	0.224	0.143	0.429

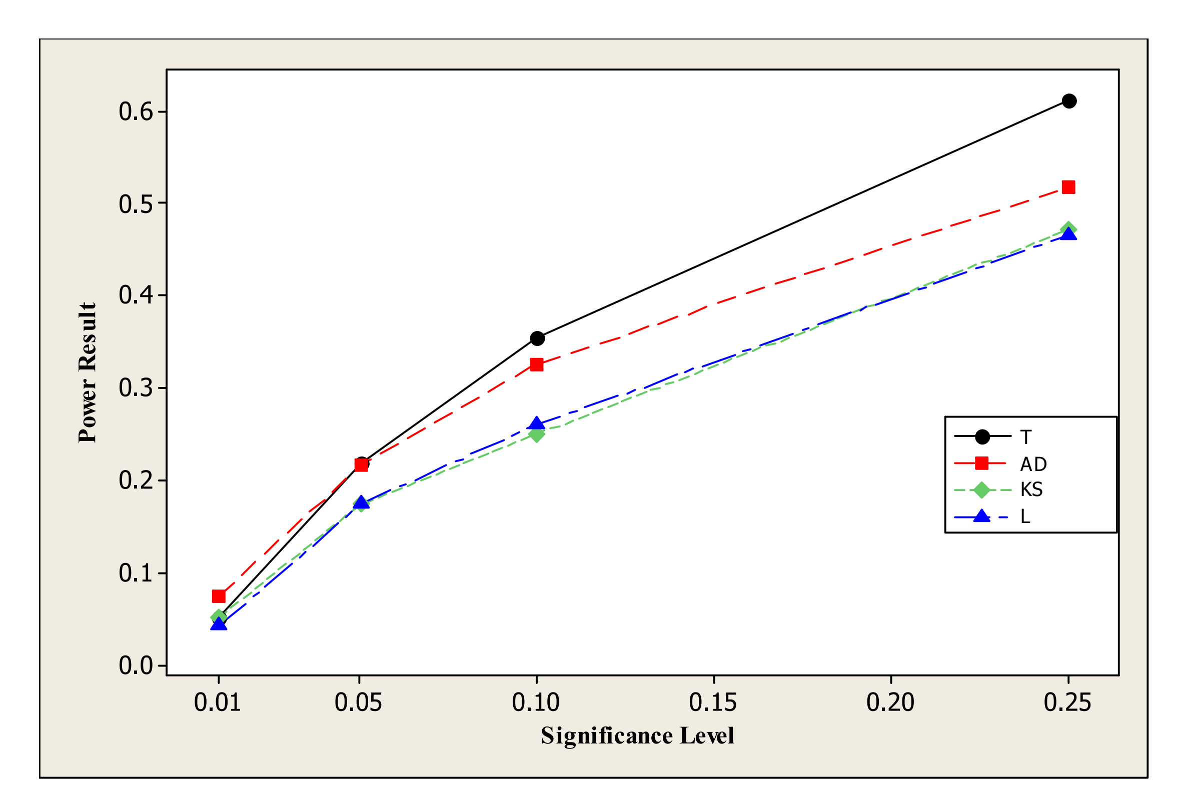


Figure 3.11 Power comparison between New test T , A^2 and KS tests ; n=20; Mle; Weibull vs. LogNormal (0,1)

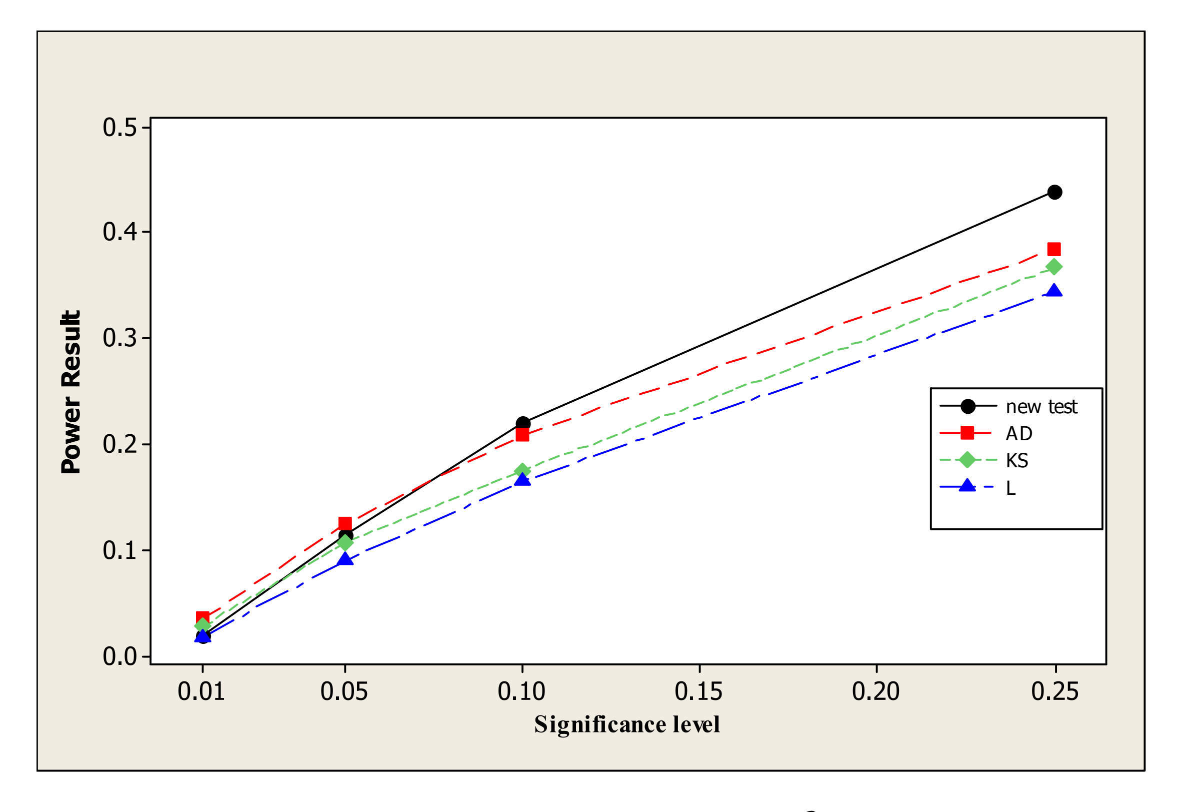


Figure 3.12 Power comparison between New test T , A^2 and KS tests ; n=20; mle ;Weibull vs. Normal (10,1)

Table 3.16 power comparisons of T test ; α = .05 ; MIX

N	test	Weibll (2,1)	Gamma (2,1)	Normal (10,1)	Lo-nom (0,1)	Cuchy (0,1)	Parto (2,1)	$\chi_{(1)}$	$\chi_{(H)}$	Average power
20	T	0.049	0.106	0.25	0.414	0.50	0.982	0.017	0.106	.303
	A^2	0.049	0.082	0.21	0.35	0.453	0.984	0.053	0.088	.283
	W^2	0.052	0.081	0.20	0.34	0.450	0.979	0.060	0.086	.281
	K-S	0.047	0.075	0.17	0.29	0.40	0.962	0.055	0.078	.259
	L	0.048	0.10	0.242	0.39	0.502	0.985	0.028	0.102	.299
30	T	0.049	0.13	0.36	0.59	0.64	0.999	0.02	0.126	.364
	A^2	0.047	0.10	0.29	0.51	0.60	0.999	0.066	0.095	.338
	W^2	0.05	0.098	0.28	0.49	0.59	0.998	0.07	0.094	.333
	K-S	0.046	0.086	0.24	0.44	0.55	0.997	0.066	0.085	.313
	L	0.05	0.12	0.34	0.56	0.65	0.999	0.039	0.117	.359
50	T	0.051	0.16	0.54	0.81	0.81	1.00	0.044	0.17	.448
	A^2	0.048	0.121	0.44	0.74	0.76	1.00	0.098	0.121	.416
	W^2	0.050	0.122	0.44	0.73	0.78	1.00	0.107	0.122	.418
	K-S	0.049	0.108	0.38	0.66	0.74	1.00	0.095	0.105	.392
	L	0.046	0.15	0.49	0.77	0.80	1.00	0.069	0.15	.434

Table 3.17 power comparisons of T test ; α = .1 ; MIX

N	test	Alternatives								
		Weibll (2,1)	Gamma (2,1)	Normal (10,1)	Lo-nom (0,1)	Cuchy (0,1)	Parto (2,1)	$\chi_{(1)}$	$\chi_{(H)}$	Average power
20	T	0.105	0.186	0.374	0.54	0.587	0.992	0.069	0.18	0.379
	A^2	0.106	0.163	0.316	0.48	0.545	0.993	0.118	0.15	0.358
	W^2	0.103	0.15	0.294	0.45	0.528	0.99	0.119	0.14	0.346
	K-S	0.102	0.145	0.274	0.42	0.504	0.983	0.123	0.138	0.336
	L	0.109	0.186	0.368	0.526	0.594	0.994	0.090	0.175	0.380
30	T	0.103	0.21	0.488	0.69	0.721	0.999	0.076	0.214	0.437
	A^2	0.101	0.17	0.407	0.62	0.68	0.999	0.125	0.176	0.409
	W^2	0.103	0.166	0.40	0.61	0.67	0.999	0.137	0.172	0.407
	K-S	0.097	0.15	0.35	0.55	0.64	0.999	0.126	0.16	0.384
	L	0.102	0.198	0.46	0.67	0.725	0.999	0.098	0.20	0.431
50	T	0.098	0.258	0.65	0.878	0.854	1.00	0.11	0.256	0.513
	A^2	0.098	0.205	0.577	0.822	0.823	1.00	0.175	0.206	0.488
	W^2	0.10	0.20	0.56	0.815	0.837	1.00	0.185	0.203	0.487
	K-S	0.103	0.184	0.51	0.762	0.802	1.00	0.174	0.186	0.465
	L	0.096	0.237	0.626	0.855	0.853	1.00	0.143	0.244	0.506

Table 3.18 power comparisons of T, : ^{0} , W^2, K and L tests ; α = .05

Test	T(mle)	T(mix)	A^2(mle)	A^2(mix)	W^2(mle)	W^2(mix)	KS(mle)	KS(mix)	L(mle)	L(mix)
Average power ,n=20	.230	.303	.243	.283	.234	.281	.217	.259	.21	.299
Average power, n=30	.283	.364	.292	.338	.277	.333	.252	.313	.284	.359
Average power ,n=50	.355	.448	.370	.416	.349	.418	.311	.392	.360	.434
Average power for n= 20,30and 50	.289	.371	.301	.345	.286	.344	.26	.321	.284	.364
Power order	7	1	6	3	8	5	10	4	9	2

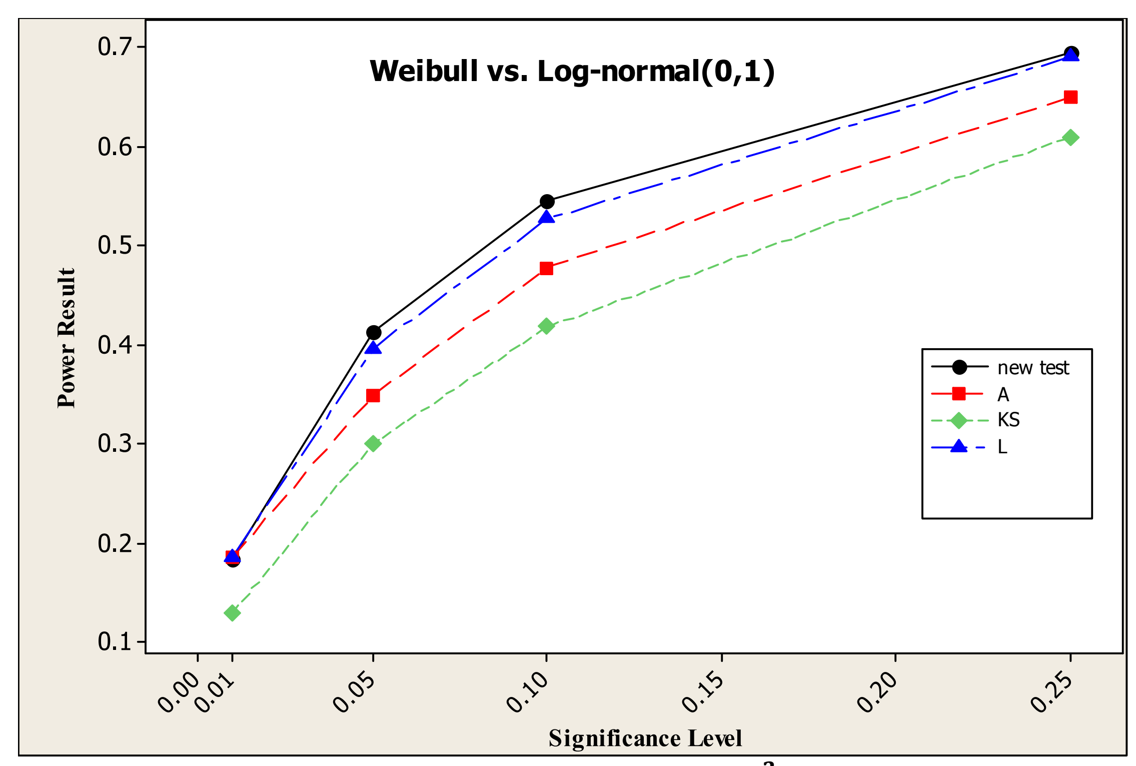


Figure 3.13 Power comparison between New test T , A^2 and KS tests ; n=20; MIX ;Weibull vs. LogNormal (0,1).

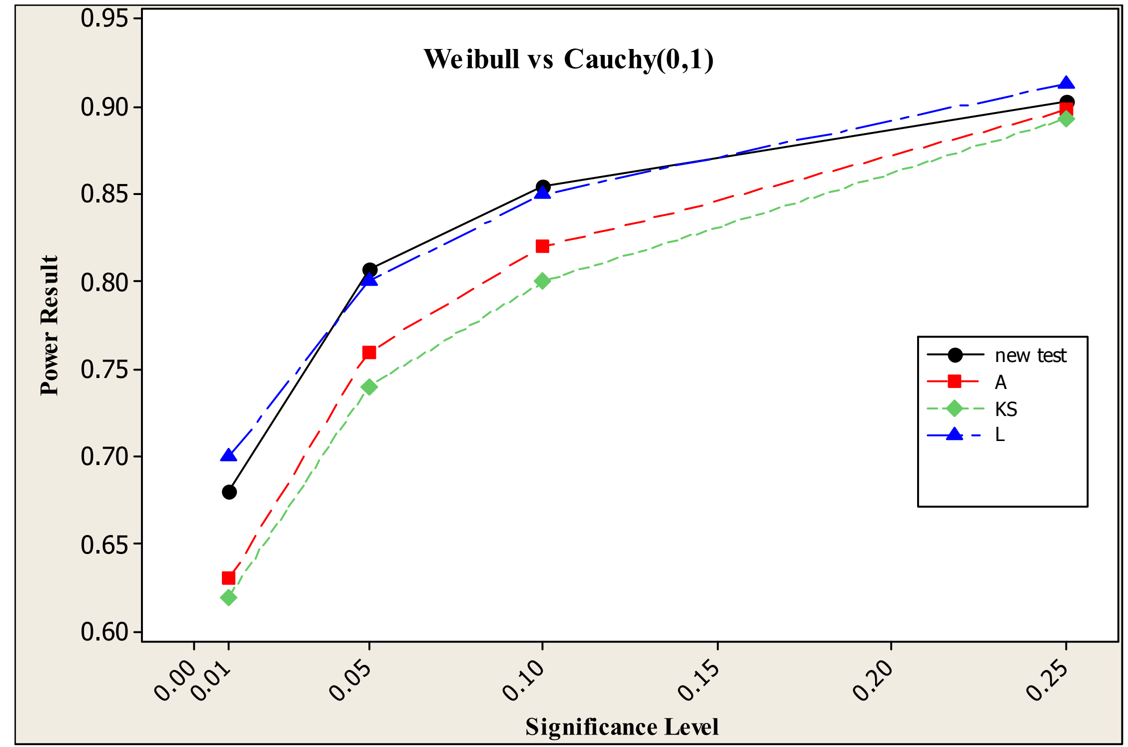


Figure 3.14 Power comparison between New test T , A^2 , KS and L tests ; n=50; MIX; Weibull vs. Cauchy (0,1).

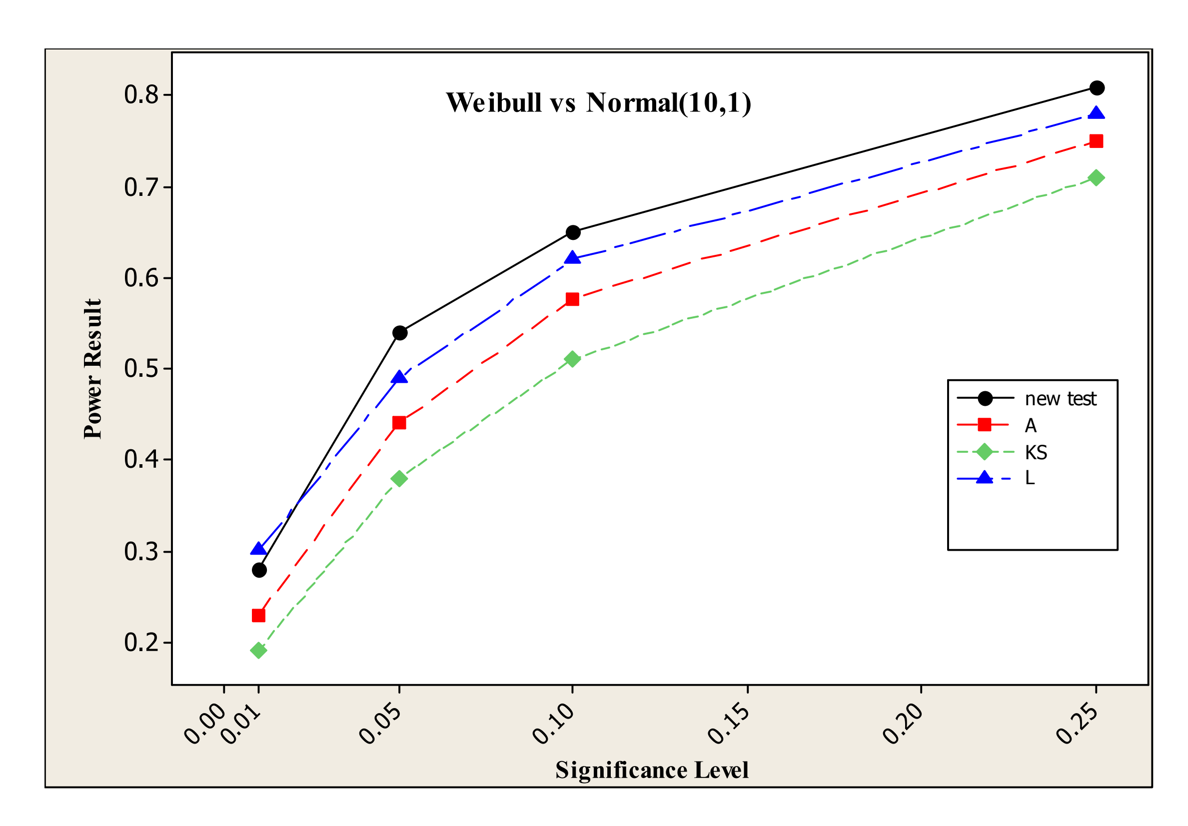


Figure 3.15 Power comparison between New test T , A^2 , KS and L tests ; n=50; mix; Weibull vs. normal (10,1).

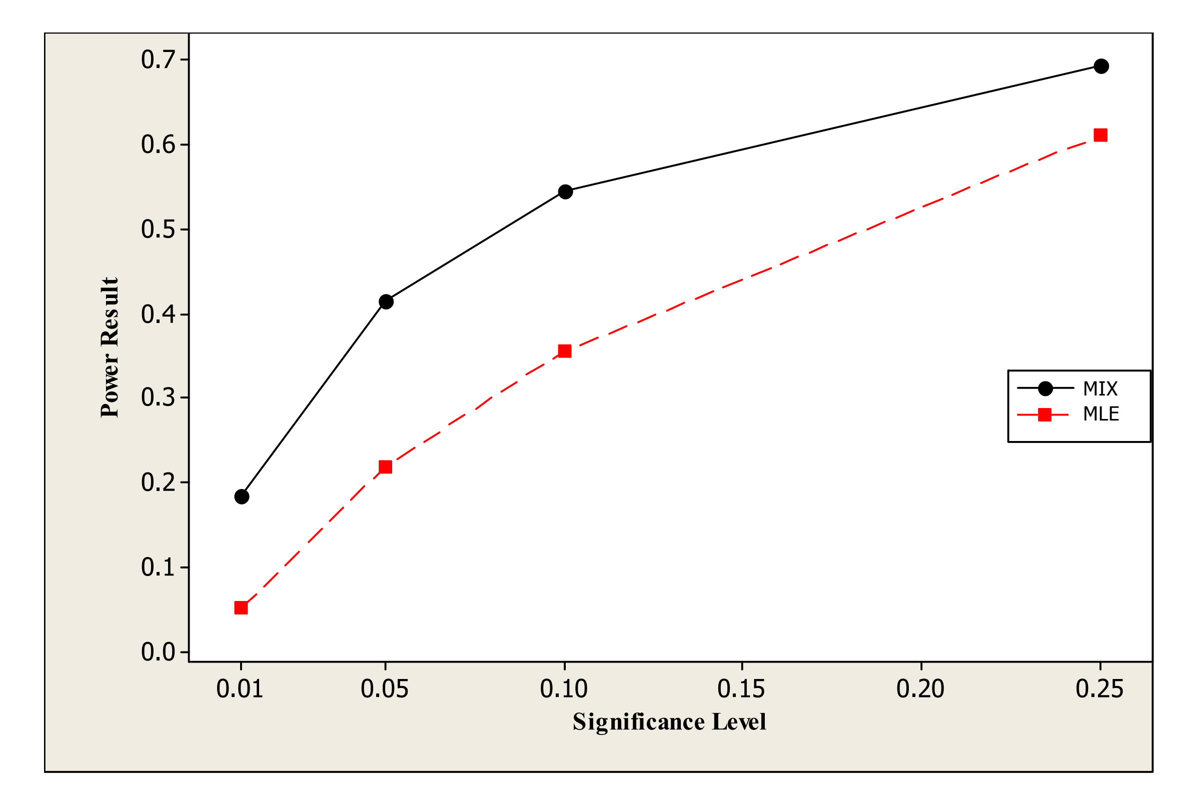


Figure 3.16 Power comparison between New test T with two method estimation n=20;α=0.05 ;Weibull vs. Log-normal (0,1).

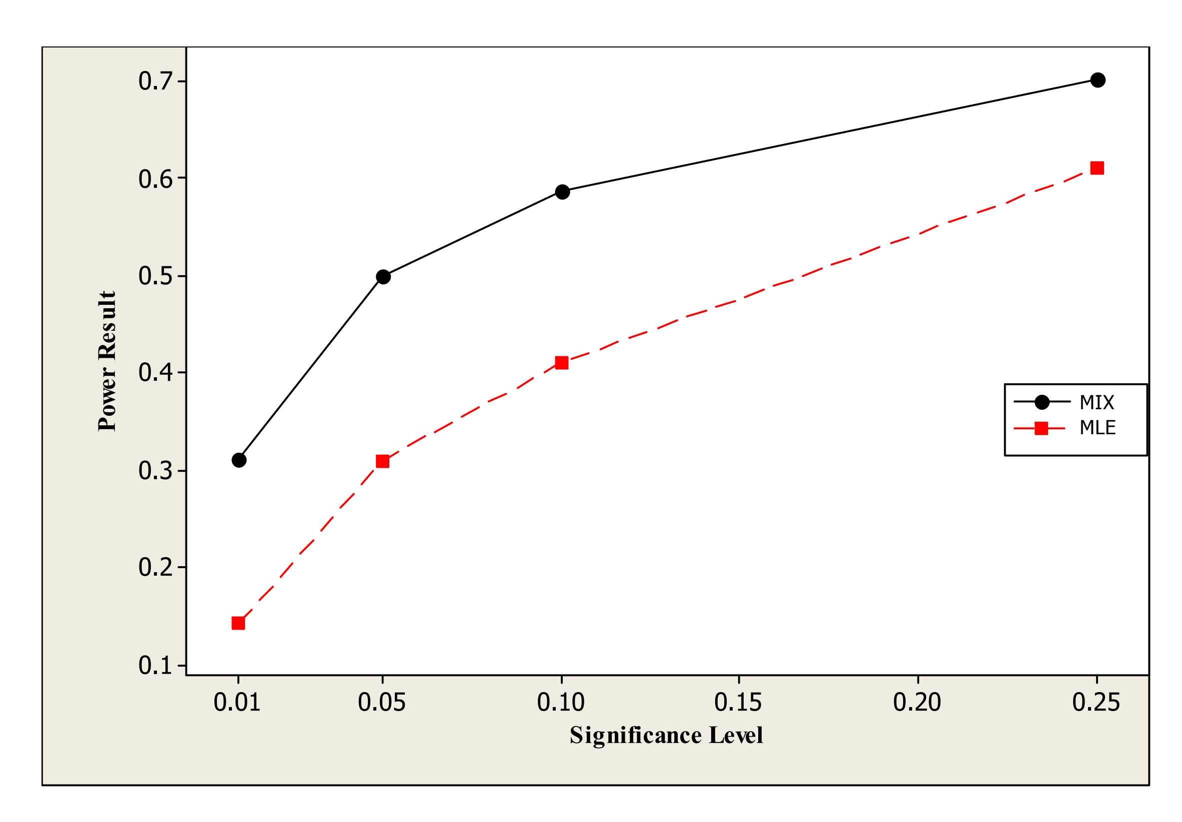


Figure 3.17 Power comparison between New test T with two method estimation n=20;α=0.05 ; Weibull vs. Cauchy (0,1).

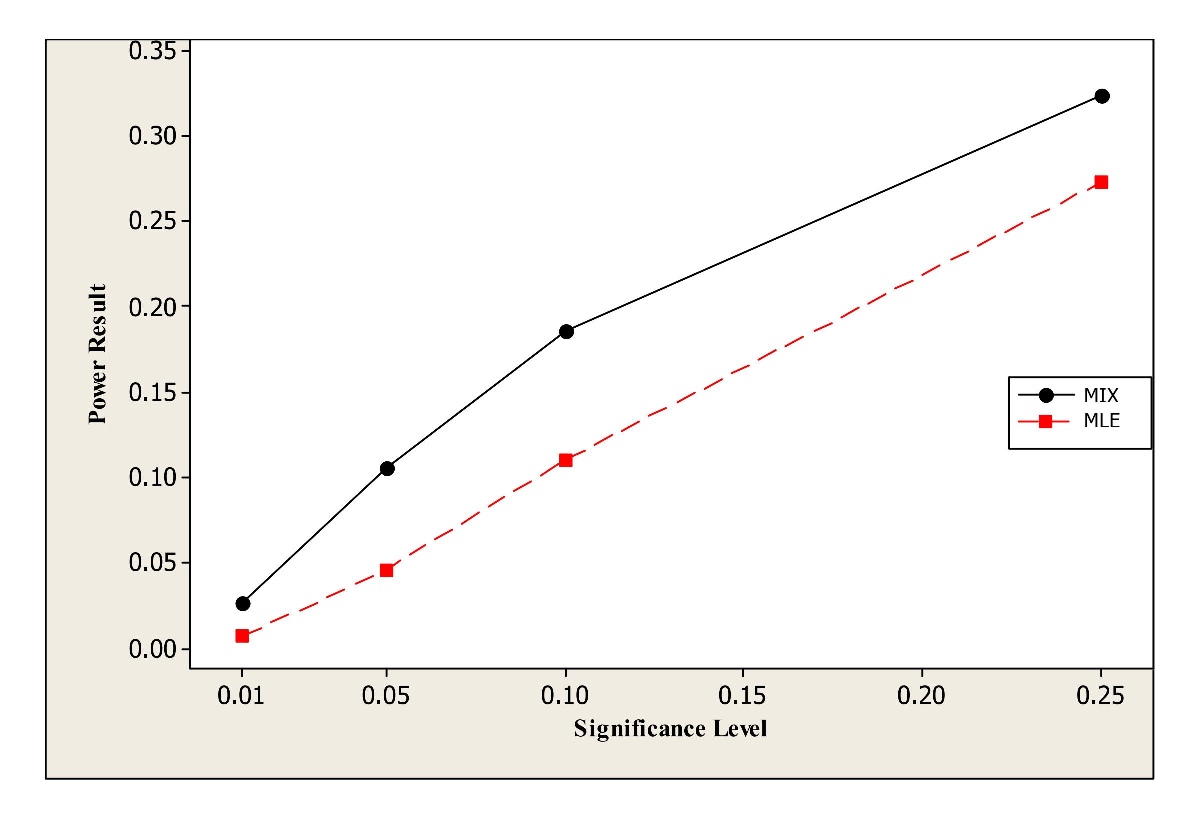


Figure 3.18 Power comparison between New test T with two method estimation

n=20;α=0.05 ; Weibull vs. Gamma (2,1)

3.6 Conclusions and Recommendations

In this thesis, we proposed a new goodness-of- fit for testing the two-parameter Weibull distribution when the shape parameter is known or unknown while the scale parameter is unknown, and must be estimated from the data. Two method of estimation has been used, first we used the maximum likelihood estimator for both parameters, second, we used a combination of both the method of moments estimator for the shape parameter and the maximum likelihood estimator for the scale parameter. Results are summarized as below:

1) The proposed test with mix method estimator (MIX) proves the highest power over all other tests for almost all alternatives under consideration.
2) For all tests under consideration T, A^2, W^2, K-S and L, the power, when the MIX method is used to estimate the Weibull parameters, is higher than that when the MLE method is used.
3) It is recommended to use a combination of both MLE and MME to estimate the parameters when using the proposed test as a goodness of fit for the Weibull distribution.

For future research the following recommendations are suggested:

- Further investigation of the MIX method of estimation and its performance compared to other methods like the MME and MLE.
- Further investigation of the distribution of the proposed test can be conducted. Wide comparisons involving more tests and larger classes of alternatives can be studied.
- Studying the power of the proposed test for truncated Weibull distribution in which case the three- parameters Weibull distribution is tested.

REFERENC

AL–Baidhani, F.A. ,Sinclar, C.D. (1987), Comparison of methods of estimation of parameters of the Weibull distribution; **Communications in Statistics Simulation and Computation** 16, 373–384.

Bain, lee J. and Max Engelhardt. (1991), **Statistical Analysis of Reliability and Life-Testing Models**(2nd ed), 115. Marcel, inc.,

Bush, J.G. ,Woodruff, B.W. , Moore, A.H. and Dunne, E.J. (1983), Modified Cramer von Mises and Anderson-Darling tests for Weibull distributions with unknown location and scale parameters. **Communications in Statistics**, Part A-Theory and Methods. 12, 2465-2476.

Cohen, A.C. and Whitten, B.J. (1988), Parameter estimation in Reliability and Life Span Models ,Marcel Dekker, New York.

Coles, S.G. (1989), On goodness–of–fit tests for the two– parameter Weibull distribution derived from the stabilized probability plot, **Biometrika** 76, 593–598
Communications in Statistics, Part A-Theory and Methods. 10:907-917.

David, F.N., Johnson, N.L. **(**1948), The probability integral transformation when parameters are estimated from the sample. Biometrika, 8, 182-190.

D'Agostino, R.B., and. Stephens, M.A.(1986), **Goodness -of-Fit Techniques**, Marcel Dekker, New York.

Dodson, B. (1994**), Weibull Analysis**; ASQC Quality Press, Milwaukee Group.LLC.

Harter, H.L. and Moore, A.H. (1965a), Maximum–likelihood estimation of the parameters of gamma and Weibull populations from complete and from censored samples, **Technometrics** 7, 639– 643.

Johnson, N.L., Kotz, S. and Balakrishnan, N. (1995), **Continuous Univariate Distributions**, Vol 2, (2nd ed). Wiley and sons New York.

Kapur, K.C. and L.R. Lamberson.(1977), **Reliability in Engineering *Design***. Wiley and sons, New York

Law,A.M. and Kelton, W.D.(1991), **Simulation Modeling and Analysis**. NYY: McGraw-Hill,inc.

Lawless, J.F. (2003), **Statistical Models and Methods for Lifetime Data**. (3rd edition), John Wiley & Sons, New York.

Liao, M. and Shimokawa,T. (1999), A new goodness-of-fit test for Type-I extreme-value and 2-parameter Weibull distributions with estimated parameters. **Journal of Statistical Computation and Simulation**, 64, 23-48.

Littel, R.C., McClave, J.T. and Offen, W.W. (1979), Goodness of fit tests for two-parameter Weibull distribution. ***Communications in Statistics***, 8(3), 257-269.

Lockhart, R.A., O'reilly, F. and Stephens, M.A. (1986), Tests for the extreme–value and Weibull distributions based on normalized spacing, **Naval Research Logistics Quarterly** 33, 413–421

Mann, N.R., Schafer, R.E. and Fertig, K.W. (1973), A goodness of fit test for the two parameter Weibull or extreme value distribution. **Communications in Statistics**, 2,383-400.

Menon, M.W. (1963), Estimation of the shape and the scale parameters of the parameters are estimated from the sample. **Biometrika**. 35,182-190.

Murthy, D.N.P. , Xie, M. and Jiang, R. (2004), **Weibull Models**, Wiley, New York

Ozturk,A., Korukoglu, S. (1988), A new test for the extreme value distribution, **Communications in *Statist***,17(4), 1375-1393.

Petrov, v.v. (1995), Limit Theorm of Probability Theory. Sequences of Independent Random Variables. Clarendon Press, Oxford.

Rinne,H. (2009),**The Weibull Distribution A Handbook**. Taylor & Francis.

Stephens, M.A. and D'Agostino ,R.D.(1986), **Goodness of Fit Techniques**. Marcel Dekker Inc.

Thoman, D.R. and Bain, L.J. (1969), Two samples tests in the Weibull distribution; **Technometrics** 11, 805–815.

Tiku, M.L. and Singh, M. (1981), Testing the two parameter Weibull distribution. Weibull distribution, **Technometrics** 5, 175–182.

Wozniak, P.J. (1994), Power of goodness of fit tests for the two–parameter Weibull distribution with estimated parameters; **Journal of Statistical Computation** & **Simulation** 50, 153–161.

Wozniak, P.J. and Li, X. (1990), Goodness-of-Fit Test for the Two Parameter Weibull Distribution with Estimated Parameters, **Journal of Statistical Computation and Simulation**, 34,133-143.

Appendex

1. Codes of Simulated quantiles of proposed test when β=2.

```
ni = 20000; β = 1; dist := WeibullDistribution[β, 1];

Do[n; Do[α; cell = {}; Do[x = RandomReal[dist, n];
   y = Sort[x];
   z = Log[y];
   m = Mean[z];
   s = StandardDeviation[z];
   θ = (Mean[y^2])^(1/2);
   w = Log[y / θ];
   s2 = StandardDeviation[w];
   m2 = Mean[w];
   R = (m2^2) / (s2^2);
   new = (R π^2 / (6 (0.57722^2)) - 1)^2;
   cell = Append[cell, new], {ni}];
  Q[α_] := Quantile[cell, α];
  Print [n, "        ", α, "        ", Q[α]], {α, 0.5, 0.95, 0.05}], {n, 10, 100, 10}]
```

2. Codes of Simulated quantiles of proposed test when the parameters are unknown and estimated by (MIX).

```
Print ["n", "        ", "α", "        ", "1-α quantile"]; n = 20; α = 0.05; ni = 20000; dist := WeibullDistribution[1, 1];
Do[n; Do[α; cell = {}; Do[x = RandomReal[dist, n];
   y = Sort[x];
   z = Log[y];
   m = Mean[z];
   s = StandardDeviation[z];
   β̂ = π / (s Sqrt[6]);
   θ̂ = (Mean[y^β̂])^(1/β̂);
   F[x_] := CDF[dist, x];
   x = (y / θ̂)^β̂;
   w = Log[y / θ̂];
s2 = StandardDeviation[w];
m2 = Mean[w];
R = (m2^2) / (s2^2);
new = (R π^2 / (6 (0.57722^2)) - 1)^2;
   cell = Append[cell, new], {ni}];
  Q[α_] := Quantile[cell, α];
  Print [n, "        ", α, "        ", Q[α]], {α, 0.5, 0.95, 0.05}], {n, 10, 100, 10}]
```

3. Codes of Simulated quantiles of proposed test when the parameters are unknown and estimated by (mle)

```
Print ["n", "        ", "α", "        ", "1-α quantile"]; n = 20; α = 0.05; ni = 20000; dist := WeibullDistribution[1, 1];
Do[n; Do[α; cell = {}; Do[x = RandomReal[dist, n];
   y = Sort[x];
   r = FindRoot[{((1/n) Sum[(x[[i]])^β, {i, 1, n}])^(1/β) == θ, n/β + Sum[Log[x[[i]]], {i, 1, n}] - n/(Sum[(x[[i]])^β, {i, 1, n}]) Sum[(x[[i]])^β Log[x[[i]]], {i, 1, n}] == 0}, {{β, 1}, {θ, 1}}];
   β̂ = β /. r; θ̂ = θ /. r;

   F[x_] := CDF[dist, x];
   x = (y / θ̂)^β̂;
   w = Log[y / θ̂];
s2 = StandardDeviation[w];
m2 = Mean[w];
R = (m2^2) / (s2^2);
new = (R π^2 / (6 (0.57722^2)) - 1)^2;
   cell = Append[cell, new], {ni}];
  Q[α_] := Quantile[cell, α];
  Print [n, "        ", α, "        ", Q[α]], {α, 0.5, 0.95, 0.05}], {n, 10, 100, 10}]
```

4. Codes of Simulated quantiles of A-D test when the parameters are unknown and estimated by (MIX).

```
Print ["n", "        ", "α", "        ", "1-α quantile"];
ni = 10000;
dist := WeibullDistribution[1, 1];
Do[n; Do[α; cell = {};
  Do[x = RandomReal[dist, n];
   y = Sort[x];
   z = Log[x];
   m = Mean[z];
   s = StandardDeviation[z];
   β̂ = π / (s Sqrt[6]);
   θ̂ = (Mean[y^β̂])^(1/β̂);
   x = (y / θ̂)^β̂;
   F[x_] := CDF[dist, x];
   NN = Range[n];
   cf = Thread[F[x]];
   n1 = Thread[Log[cf]];
   n2 = Thread[Log[1 - cf]];

   and = -n - ((2 NN - 1).n1 + (2 n + 1 - 2 NN).n2) / n;
   cell = Append[cell, and], {ni}];
  Q[α_] := Quantile[cell, α];
  Print [n, "        ", α, "        ", Q[α]], {α, 0.5, 0.95, 0.05}], {n, 10, 100, 10}]
```

5. Codes of Simulated quantiles of A-D test when the parameters are unknown and estimated by (MLE).

```
Print ["n", "        ", "α", "        ", "1-α quantile"];
ni = 10 000;
dist := WeibullDistribution[1, 1];
Do[n; Do[α; cell = {};
  Do[x = RandomReal[dist, n];
   y = Sort[x];
   r = FindRoot[{((1/n) Sum[(x[[i]])^β, {i, 1, n}])^(1/β) == θ, n/β + Sum[Log[x[[i]]], {i, 1, n}] - n/(Sum[(x[[i]])^β, {i, 1, n}]) Sum[(x[[i]])^β Log[x[[i]]], {i, 1, n}] == 0}, {{β, 1}, {θ, 1}}];
   β̂ = β /. r; θ̂ = θ /. r;
   x = (y / θ̂)^β̂;
   F[x_] := CDF[dist, x];
   NN = Range[n];
   ecf1 = NN / n;
   ecf2 = (NN - 1) / n;
   cf = Thread[F[x]];
   n1 = Thread[Log[cf]];
   n2 = Thread[Log[1 - cf]];
   and = -n - ((2 NN - 1).n1 + (2 n + 1 - 2 NN).n2) / n;
   cell = Append[cell, CvM], {ni}];
  Q[α_] := Quantile[cell, α];
  Print [n, "        ", α, "        ", Q[α]], {α, 0.5, 0.95, 0.05}], {n, 10, 100, 10}]
```

---

6. Codes of Simulated quantiles K-S test when the parameters are unknown and estimated by (MIX).

```
Print ["n", "        ", "α", "        ", "1-α quantile"];
ni = 20 000;
dist := WeibullDistribution[1, 1];
Do[n; Do[α; cell = {};
  Do[x = RandomReal[dist, n];
   y = Sort[x];
   z = Log[x];
   m = Mean[z];
   s = StandardDeviation[z];
   β̂ = π / (s Sqrt[6]);
   θ̂ = (Mean[y^β̂])^(1/β̂);
   F[x_] := CDF[dist, x];
   x = (y / θ̂)^β̂;
   NN = Range[n];
   ecf1 = NN / n;
   ecf2 = (NN - 1) / n;
   cf = Thread[F[x]];
   kol1 = Max[ecf1 - cf];
   kol2 = Max[cf - ecf2];
   kol = Max[kol1, kol2];
   cell = Append[cell, kol], {ni}];
  Q[α_] := Quantile[cell, α];
  Print [n, "        ", α, "        ", Q[α]], {α, 0.75, 0.95, 0.05}], {n, 10, 100, 10}]
```

7. Codes of Simulated quantiles of C-M test when the parameters are unknown and estimated by (MIX).

```
Print ["n", "       ", "α", "       ", "1-α quantile"];
ni = 20000;
dist := WeibullDistribution[1, 1];
Do[n; Do[α; cell = {};
  Do[x = RandomReal[dist, n];
   y = Sort[x];
   z = Log[x];
   m = Mean[z];
   s = StandardDeviation[z];
   β̂ = π / (s Sqrt[6]);
   θ̂ = (Mean[y^β̂])^(1/β̂);
   x = (y / θ̂)^β̂;
   F[x_] := CDF[dist, x];
   NN = Range[n];
   ecf1 = NN / n;
   ecf2 = (NN - 1) / n;
   cf = Thread[F[x]];
   CvM = 1/(12 n) + Sum[((2 i - 1)/(2 n) - cf[[i]])^2, {i, 1, n}];
   cell = Append[cell, CvM], {ni}];
  Q[α_] := Quantile[cell, α];
  Print [n, "       ", α, "       ", Q[α]], {α, 0.75, 0.95, 0.05}], {n, 10, 100, 10}]
```

8. Codes of Simulated quantiles of L test when the parameters are unknown and estimated by (MIX).

```
Print ["n", "      ", "α", "      ", "1-α quantile"];
ni = 20 000;
dist := WeibullDistribution[1, 1];
Do[n; Do[α; cell = {};
  Do[x = RandomReal[dist, n];
   y = Sort[x];
   z = Log[x];
   m = Mean[z];
   s = StandardDeviation[z];
   β̂ = π / (s Sqrt[6]);
   θ̂ = (Mean[y^β̂])^(1/β̂);
   F[x_] := CDF[dist, x];
   x = (y / θ̂)^β̂;
   NN = Range[n];
   ecf1 = NN / n;
   ecf2 = (NN - 1) / n;
   cf = Thread[F[x]];
   LiShim = 1/Sqrt[n] Sum[Max[ecf1[[i]] - cf[[i]], cf[[i]] - ecf2[[i]]] / Sqrt[(cf[[i]]) (1 - cf[[i]])], {i, 1, n}];
   cell = Append[cell, LiShim], {ni}];
  Q[α_] := Quantile[cell, α];
  Print [n, "      ", α, "      ", Q[α]], {α, 0.75, 0.95, 0.05}], {n, 10, 100, 10}]
```

9. Codes of Simulated quantiles of L test when the parameters are unknown and estimated by (MLE).

```
Print ["n", "      ", "α", "      ", "1-α quantile"];
ni = 1000;
dist := WeibullDistribution[1, 1];
Do[n; Do[α; cell = {};
  Do[x = RandomReal[dist, n];
   y = Sort[x];
   r = FindRoot[{(1/n Sum[(x[[i]])^β, {i, 1, n}])^(1/β) == θ, n/β + Sum[Log[x[[i]]], {i, 1, n}] - n/Sum[(x[[i]])^β, {i, 1, n}] Sum[(x[[i]])^β Log[x[[i]]], {i, 1, n}] == 0}, {{β, 1}, {θ, 1}}]; β̂ = β /. r; θ̂ = θ /. r;
   x = (y / θ̂)^β̂;
   F[x_] := CDF[dist, x];
   NN = Range[n];
   ecf1 = NN / n;
   ecf2 = (NN - 1) / n;
   cf = Thread[F[x]];
   LiShim = 1/Sqrt[n] Sum[Max[ecf1[[i]] - cf[[i]], cf[[i]] - ecf2[[i]]] / Sqrt[(cf[[i]]) (1 - cf[[i]])], {i, 1, n}];
   cell = Append[cell, LiShim], {ni}];
  Q[α_] := Quantile[cell, α];
  Print [n, "      ", α, "      ", Q[α]], {α, 0.99, 0.99, 0.05}], {n, 10, 100, 10}]
```

10. Codes of Simulated quantiles of C-M test when the parameters are unknown and estimated by (MLE).

```
Print ["n", "        ", "α", "        ", "1-α quantile"];
ni = 10;
dist := WeibullDistribution[1, 1];
Do[n; Do[α; cell = {};
  Do[x = RandomReal[dist, n];
   y = Sort[x];
   r = FindRoot[{((1/n) Sum[(x[[i]])^β, {i, 1, n}])^(1/β) == θ, n/β + Sum[Log[x[[i]]], {i, 1, n}] - n/(Sum[(x[[i]])^β, {i, 1, n}]) Sum[(x[[i]])^β Log[x[[i]]], {i, 1, n}] == 0}, {{β, 1}, {θ, 1}}];
   β̂ = β /. r; θ̂ = θ /. r;
   x = (y / θ̂)^β̂;
   F[x_] := CDF[dist, x];
   NN = Range[n];
   ecf1 = NN / n;
   ecf2 = (NN - 1) / n;
   cf = Thread[F[x]];
   NN = Range[n];
   ecf1 = NN / n;
   ecf2 = (NN - 1) / n;
   cf = Thread[F[x]];
   CvM = 1/(12 n) + Sum[((2 i - 1)/(2 n) - cf[[i]])^2, {i, 1, n}];
   cell = Append[cell, CvM], {ni}];
  Q[α_] := Quantile[cell, α];
  Print [n, "        ", α, "        ", Q[α]], {α, 0.5, 0.95, 0.05}], {n, 10, 100, 10}]
```

11. Codes of Simulated quantiles of K-S test when the parameters are unknown and estimated by (MLE).

```
Print ["n", "        ", "α", "        ", "1-α quantile"];
ni = 20000;
dist := WeibullDistribution[1, 1];
Do[n; Do[α; cell = {};
  Do[x = RandomReal[dist, n];
   y = Sort[x];
   r = FindRoot[{((1/n) Sum[(x[[i]])^β, {i, 1, n}])^(1/β) == θ, n/β + Sum[Log[x[[i]]], {i, 1, n}] - n/(Sum[(x[[i]])^β, {i, 1, n}]) Sum[(x[[i]])^β Log[x[[i]]], {i, 1, n}] == 0}, {{β, 1}, {θ, 1}}];
   β̂ = β /. r; θ̂ = θ /. r;
   x = (y / θ̂)^β̂;
   F[x_] := CDF[dist, x];
   NN = Range[n];
   ecf1 = NN / n;
   ecf2 = (NN - 1) / n;
   cf = Thread[F[x]];
   kol1 = Max[ecf1 - cf];
   kol2 = Max[cf - ecf2];
   kol = Max[kol1, kol2];
   cell = Append[cell, kol], {ni}];
  Q[α_] := Quantile[cell, α];
  Print [n, "        ", α, "        ", Q[α]], {α, 0.95, 0.95, 0.05}], {n, 10, 10, 10}]
```

12. Codes of Simulated Power of all tests when shape parameter is known (β=2).

```
n = 20; α = .05; ni = 10 000; dist := WeibullDistribution[1, 1]; dist1 := WeibullDistribution[4, 1];
cell1 = {}; cut1 = 0.234;
cell2 = {}; cut2 = 0.220;
cell3 = {}; cut3 = 1.275;
cell4 = {}; cut4 = 1.43;
cell5 = {}; cut5 = 0.290;
Do[x = RandomReal[dist1, n];
  y = Sort[x];
  z = Log[y];
  m = Mean[z];
  s = StandardDeviation[z];
  θ̂ = (Mean[y^2])^(1/2);
  x = (y / θ̂)^2;
  F[x_] := CDF[dist, x];
  NN = Range[n];
  ecf1 = NN / n;
  ecf2 = (NN - 1) / n;
  cf = Thread[F[x]];
  kol1 = Max[ecf1 - cf];
  kol2 = Max[cf - ecf2];
  kol = Max[kol1, kol2]; "test1";
  CvM = 1/(12 n) + Sum[((2 i - 1)/(2 n) - cf[[i]])^2, {i, 1, n}]; "test2";
  n1 = Thread[Log[cf]];
  n2 = Thread[Log[1 - cf]];
  and = -n - ((2 NN - 1).n1 + (2 n + 1 - 2 NN).n2)/n; "test3";

  LiShim = 1/Sqrt[n] Sum[Max[ecf1[[i]] - cf[[i]], cf[[i]] - ecf2[[i]]]/Sqrt[(cf[[i]]) (1 - cf[[i]])], {i, 1, n}]; "test4";
  w = Log[y / θ̂];
  s2 = StandardDeviation[w];
  m2 = Mean[w];
  R = (m2^2) / (s2^2);
  new = (R π^2 / (6 (0.57722^2)) - 1)^2; "test5";
  test1 = If[kol < cut1, 0, 1];
  test2 = If[CvM < cut2, 0, 1];
  test3 = If[and < cut3, 0, 1];
  test4 = If[LiShim < cut4, 0, 1];
  test5 = If[new < cut5, 0, 1];
  cell1 = Append[cell1, test1];
  cell2 = Append[cell2, test2];
  cell3 = Append[cell3, test3];
  cell4 = Append[cell4, test4];
  cell5 = Append[cell5, test5], {ni}];
Print["n        ", "α      ", "Kol      ", "CvM     ", "A-D     ", " Shim      ", "new     "];
Print[
 n, "     ",
 α, "     ",
 N[Total[cell1] / ni], "     ",
 N[Total[cell2] / ni], "     ",
 N[Total[cell3] / ni], "     ",
 N[Total[cell4] / ni], "     ",
 N[Total[cell5] / ni]]
```

13. Codes of Simulated Power of all tests when both parameters are unknown and estimated by(MLE).

```
n = 50; ni = 10 000; dist := WeibullDistribution[1, 1]; dist1 : ChiSquareDistribution[1];
cell1 = {}; cut1 = 0.113;
cell2 = {}; cut2 = 0.089;
cell3 = {}; cut3 = 0.625;
cell5 = {}; cut5 = 0.887;
cell6 = {}; cut6 = 0.025;
Do[x = RandomReal[dist1, n]; x = Abs[x];
  y = Sort[x];
  r = FindRoot[{((1/n) Sum[(x[[i]])^β, {i, 1, n}])^(1/β) == θ, n/β + Sum[Log[x[[i]]], {i, 1, n}] - n/(Sum[(x[[i]])^β, {i, 1, n}]) Sum[(x[[i]])^β Log[x[[i]]], {i, 1, n}] == 0}, {{β, 1}, {θ, 1}}];
  β̂ = β /. r; θ̂ = θ /. r;
  x = (y / θ̂)^β̂;
  F[x_] := CDF[dist, x];
  NN = Range[n];
  ecf1 = NN / n;
  ecf2 = (NN - 1) / n;
  cf = Thread[F[x]];
  kol1 = Max[ecf1 - cf];
  kol2 = Max[cf - ecf2];
  kol = Max[kol1, kol2]; "test1";
  CvM = 1/(12 n) + Sum[((2 i - 1)/(2 n) - cf[[i]])^2, {i, 1, n}]; "test2";
  n1 = Thread[Log[cf]];
  n2 = Thread[Log[1 - cf]];
  and = -n - ((2 NN - 1).n1 + (2 n + 1 - 2 NN).n2)/n; "test3";

  LiShim = 1/Sqrt[n] Sum[Max[ecf1[[i]] - cf[[i]], cf[[i]] - ecf2[[i]]]/Sqrt[(cf[[i]]) (1 - cf[[i]])], {i, 1, n}]; "test5";
  w = Log[y / θ̂];
  s2 = StandardDeviation[w];
  m2 = Mean[w];
  R = (m2^2) / (s2^2);
  new = (R π^2 / (6 (0.57722^2)) - 1)^2; "test6";
  test1 = If[kol < cut1, 0, 1];
  test2 = If[CvM < cut2, 0, 1];
  test3 = If[and < cut3, 0, 1];
  test5 = If[LiShim < cut5, 0, 1];
  test6 = If[new < cut6, 0, 1];
  cell1 = Append[cell1, test1];
  cell2 = Append[cell2, test2];
  cell3 = Append[cell3, test3];
  cell5 = Append[cell5, test5];
  cell6 = Append[cell6, test6], {ni}];
Print["n      ", "α      ", "Kol     ", "CvM      ", "A-D     ", " Shim     ", "new    "];
Print[n, "    ",
 0.1, "    ",
 N[Total[cell1] / ni], "    ",
 N[Total[cell2] / ni], "    ",
 N[Total[cell3] / ni], "    ",
 N[Total[cell5] / ni], "    ",
 N[Total[cell6] / ni]]
```

14. Codes of Simulated Power of all tests when both parameters are unknown and estimated by (MIX).

```
n = 20; α = 0.1; ni = 10 000; dist := WeibullDistribution[1, 1]; dist1 := ParetoDistribution[2, 1];
cell1 = {}; cut1 = 0.216;
cell2 = {}; cut2 = 0.179;
cell3 = {}; cut3 = .951;
cell5 = {}; cut5 = 1.196;
cell6 = {}; cut6 = 0.266;
Do[x = RandomReal[dist1, n];
  x = Abs[x];
  y = Sort[x];
  z = Log[y];
  m = Mean[z];
  s = StandardDeviation[z];
  β̂ = π / (s Sqrt[6]);
  θ̂ = (Mean[y^β̂])^(1/β̂);
  x = (y / θ̂)^β̂;
  F[x_] := CDF[dist, x];
  NN = Range[n];
  ecf1 = NN / n;
  ecf2 = (NN - 1) / n;
  cf = Thread[F[x]];
  kol1 = Max[ecf1 - cf];
  kol2 = Max[cf - ecf2];
  kol = Max[kol1, kol2]; "test1";
  CvM = 1/(12 n) + Sum[((2 i - 1)/(2 n) - cf[[i]])^2, {i, 1, n}]; "test2";
  n1 = Thread[Log[cf]];
  n2 = Thread[Log[1 - cf]];
  and = -n - ((2 NN - 1).n1 + (2 n + 1 - 2 NN).n2)/n; "test3";

  LiShim = 1/Sqrt[n] Sum[Max[ecf1[[i]] - cf[[i]], cf[[i]] - ecf2[[i]]]/Sqrt[(cf[[i]]) (1 - cf[[i]])], {i, 1, n}]; "test5";
  w = Log[y / θ̂];
  s2 = StandardDeviation[w];
  m2 = Mean[w];
  R = (m2^2) / (s2^2);
  new = (R π^2 / (6 (0.57722^2)) - 1)^2; "test6";
  test1 = If[kol < cut1, 0, 1];
  test2 = If[CvM < cut2, 0, 1];
  test3 = If[and < cut3, 0, 1];
  test5 = If[LiShim < cut5, 0, 1];
  test6 = If[new < cut6, 0, 1];
  cell1 = Append[cell1, test1];
  cell2 = Append[cell2, test2];
  cell3 = Append[cell3, test3];
  cell5 = Append[cell5, test5];
  cell6 = Append[cell6, test6], {ni}];
Print["n        ", "α        ", "Kol       ", "CvM        ", "A-D       ", " Shim       ", "new     "];
Print[
 n, "     ",
 α, "     ",
 N[Total[cell1] / ni], "     ",
 N[Total[cell2] / ni], "     ",
 N[Total[cell3] / ni], "     ",
 N[Total[cell5] / ni], "     ",
 N[Total[cell6] / ni]]
```

اختبار حسن المطابقة لتوزيع ويبل

إعداد
عبدالمجيد القليطي

المشرف
الدكتور. أحمد الزغول

ملخص

في هذه الرسالة، تم اقتراح اختباراً جديداً لحسن مطابقة البيانات لتوزيع ويبل ،وقد أخذ بعين الإعتبار حالتين لمعلمة الشكل حينما تكون معلومة أو مجهولة، وقد استخدم لتقديرها طريقة الإماكن الأ عظم كما تم استخدام طريقة العزوم في التقدير. استخدمت محاكاة مونت كارلو لحساب القيم الحرجة للتوزيع عند مستويات دلالة مختلفة ولحجوم عينات متعددة.
ولقياس كفاءة التوزيع قمنا بحساب قوة التوزيع عندما تكون الفرضية البديلة مجموعة مختلفة من التوزيعات تتضمن : التوزيع الطبيعي،لوغاريتم التوزيع الطبيعي، توزيع مربع-كاي، توزيع كوشي ، توزيع باريتو، توزيع جاما وغيرها من التوزيعات.
ولمقارنة كفاءة الاختبار المقترح تمت مقارنة قوة الاختبار مع مجموعة من الاختبارات المعروفة وهي: اختبار كولموجورف - سميرنوف، اختبار أندرسون- دارلينق، اختبار كريمر – فان ميس و اختبار ليو- شيموكوا.
أظهرت النتائج أن الأختبار المقترح منافس جيد للإختبارت السابقة في مجموعة التوزيعات التي أخذت بعين الإعتبار.